DRAFT: Three Intellectual Temperaments: Birds, Frogs and Beavers

by **multifoliaterose**30th Oct

Personal Blog

Here is a draft of a potential top-level post which I'd welcome feedback on. I would appreciate any suggestions, corrections, additional examples, qualifications, or refinements.

Birds, Frogs and Beavers

The introduction of Birds and Frogs by Freeman Dyson reads

Some mathematicians are birds, others are frogs. Birds fly high in the air and survey broad vistas of mathematics out to the far horizon. They delight in concepts that unify our thinking and bring together diverse problems from different parts of the landscape. Frogs live in the mud below and see only the flowers that grow nearby. They delight in the details of particular objects, and they solve problems one at a time. I happen to be a frog, but many of my best friends are birds. The main theme of my talk tonight is this. Mathematics needs both birds and frogs. Mathematics is rich and beautiful because birds give it broad visions and frogs give it intricate details. Mathematics is both great art and important science, because it combines generality of concepts with depth of structures. It is stupid to claim that birds are better than frogs because they see farther, or that frogs are better than birds because they see deeper. The world of mathematics is both broad and deep, and we need birds and frogs working together to explore it.

Dyson is far from the first to have categorized mathematicians in such a fashion. For example, in The Two Cultures of Mathematics British mathematician Timothy Gowers wrote

The "two cultures" I wish to discuss will be familiar to all professional mathematicians. Loosely speaking, I mean the distinction between mathematicians who regard their central aim as being to solve problems, and those who are more concerned with building and understanding theories. This difference of attitude has been remarked on by many people, and I do not claim any credit for noticing it. As with most categorizations, it involves a certain oversimplication, but not so much as to make it useless. If you are unsure to which class you belong, then consider the following two statements.

(i) The point of solving problems is to understand mathematics better.
(ii) The point of understanding mathematics is to become better able to solve problems.

Most mathematicians would say that there is truth in both (i) and (ii). Not all problems are equally interesting, and one way of distinguishing the more interesting ones is to demonstrate that they improve our understanding of mathematics as a whole. Equally, if somebody spends many years struggling to understand a difficult area of mathematics, but does not actually do anything with this understanding, then why should anybody else care? However, many, and perhaps most, mathematicians will not agree equally strongly with the two statements.

Similarly, Gian Carlo Rota's candid Indiscrete Thoughts contains an essay titled *Problem Solvers and Theorizers* which draws a similar dichotomy:

Mathematicians can be subdivided into two types: problem solvers and theorizer. Most mathematicians are a mixture of the two although it is easy to find extreme examples of both types.

To the problem solver, the supreme achievement in mathematics is the solution to a problem that had been given up as hopeless. It matters little that the solution may be clumsy; all that counts is that it should be the first and that the proof be correct. Once the problem solver finds the new solution, he will permanently lose interest in it, and will listen to new and simplified proofs with an air of condescension and boredom.

The problem solver is a conservative at heart. For him, mathematics consists of a sequence of challenges to be met, an obstacle course of problems. The mathematical concepts required to state mathematical problems are tacitly assumed to be eternal and immutable.

Mathematical exposition is regarded as an inferior undertaking. New theories are viewed with deep suspicion, as intruders who must prove their worth by posing challenging problems before they can gain attention. The problem solver resents generalizations, especially those that may succeed in trivializing the solution to one of his problems.

The problem solver is the role model for budding young mathematicians. When we describe to the public the conquests of mathematics, our shining heroes are the problem solvers.

To the theorizer, the supreme achievement of mathematics is a theory that sheds sudden light on some incomprehensible phenomenon. Success in mathematics does not lie in solving problems but in their trivialization. The moment of glory comes with the discovery of a new theory that does not solve any of the old problems but renders them irrelevant.

The theorizer is a revolutionary at heart. Mathematical concepts received from the past are regarded as imperfect instances of more general ones yet to be discovered.

Mathematical exposition is considered a more difficult undertaking than mathematical research.

To the theorizer, the only mathematics that will survive are the definitions. Great definitions are what mathematics contributes to the world. Theorems are tolerated as a necessary evil since they play a supporting role - or rather, as the theorizer will reluctantly admit, an essential role - in the understanding of the definitions.

Theorizers often have trouble being recognized by the community of mathematicians. Their consolation is the certainty, which may or may not be borne out by history, that their theories will survive long after the problems of the day have been forgotten.

If I were a space engineer looking for a mathematician to help me send a rocket into space, I would choose a problem solver. But if I were looking for a mathematician to give a good education to my child, I would unhesitatingly prefer a theorizer.

I believe that Rota's characterizations of problem solvers and theorizers are exaggerated but nevertheless in the right general direction. Rota's remarks are echoed in Colin McLarty's: The Rising Tide: Grothendieck on simplicity and generality

Grothendieck describes two styles in mathematics. If you think of a theorem to be proved as a nut to be opened, so as to reach "the nourishing flesh protected by the shell", then the hammer and chisel principle is: "put the cutting edge of the chisel against the shell and strike hard. If needed, begin again at many different points until the shell cracks—and you are satisfied". He says:

"I can illustrate the second approach with the same image of a nut to be opened. The first analogy that came to my mind is of immersing the nut in some softening liquid, and why not simply water? From time to time you rub so the liquid penetrates better, and otherwise you let time pass. The shell becomes more flexible through weeks and months—when the time is ripe, hand pressure is enough, the shell opens like a perfectly ripened avocado!

A different image came to me a few weeks ago. The unknown thing to be known appeared to me as some stretch of earth or hard marl, resisting penetration. . . the sea advances insensibly in silence, nothing seems to happen, nothing moves, the water is so far off you hardly hear it. . . yet it finally surrounds the resistant substance."

[...]

Deligne describes a characteristic Grothendieck proof as a long series of trivial steps where "nothing seems to happen, and yet at the end a highly non-trivial theorem is there."

In addition to the sources cited above, Grothendieck discusses a dichotomy which resembles that of birds and frogs in the section of *Recoltes et Semailles* titled *The Inheritors and the Builders* and Lee Smolin discusses such a dicotomy in The Trouble With Physics Chapter 18.

In his Opinion 95, Doron Zeilberger added a supplement to Dyson's classification, saying:

I agree that both frogs and birds are crucial for the progress of science, but, even more important, for the progress of mathematics in the computer age, is the **beaver**, who will build the needed infrastructre of computer mathematics, that would eventually enable us to solve many outstanding open problems, and many new ones. Consequently, the developers of computer algebra systems, and creators of algorithms, are even more important than both birds and frogs.

Zeilberger's statement that beavers are more important for the process of science than birds and frogs is debatable and I do not endorse it; but I believe that Zeilberger is correct to identify a third category consisting of people whose primary interest is in algorithms. Indeed, as Laurens Gunnarsen recently pointed out to me, Felix Klein had already identified such a category in his 1908 lectures on Elementary Mathematics from an Advanced Standpoint: Arithmetic, Algebra and Analysis. In the section titled *Concerning the Modern Development and the General Structure of Mathematics*, Klein identified three plans A, B, and C roughly corresponding to the natural activities of frogs, birds and beavers respectively:

…we might say that Plan A is based on a more particularistic conception of science which divides the total field into a series of mutually separated parts and attempts to develop each part for itself, with a minimum of resources and with all possible avoidance of borrowing from neighboring fields. Its ideal is to crystallize out each of the partial fields into a logically closed system. On the contrary, the supporter of Plan B lays the chief stress upon the organic combination of the partial fields, and upon the stimulation which these exert one upon another. He prefers, therefore, the methods which open for him an understanding of several fields under a uniform point of view. His ideal is the comprehension of the sum total of mathematical science as a great connected whole.

[…]

For a complete understanding of the development of mathematics, we must, however, think of a still third plan C, which, along side of and within the processes of development A and B, often plays an important role. It has to do with a method which one denotes by the word algorithm, derived from a mutilated form of an Arabian mathematician. All ordered formal calculation is, at bottom, algorithmic, in particular, the calculation with letters is an algorithm. We have repeatedly emphasized what an important part in the development of the science has been played by the algorithmic process, as a quasi-independent, onward-driving force, inherent in the formulas, operating apart from the intension and insight of the mathematician, at the time, often in opposite to them. In the beginning of the infinitesimal calculus, as we shall see later on, the algorithm has often forced new notions and operations, even before one could justify their admissibility. Even at higher levels of the development, these algorithmic considerations can be, and actually have been, very fruitful, so that one can justly call them the groundwork of mathematical development. We must then completely ignore history, if, as is sometimes done today, we cast these circumstances contemptuously aside as mere "formal" developments.

The three categories described above appear to have correlates of personality traits, mathematical interests and superficially nonmathematical interests. Below I'll engage in some speculation about this.

Correlates of the bird category

My impression is that birds tend to have high openness to experience, be anti-conformist, highly emotional sensitivity, and interested in high art, history, philosophy, religion and geometry. Here I'll give some supporting evidence. I believe that the thinkers discussed would identify themselves as birds.

1. Dyson's article discusses Yuri Manin as follows:

The book is mainly about mathematics. It may come as a surprise to Western readers that he writes with equal eloquence about other subjects such as the collective unconscious, the origin of human language, the psychology of autism, and the role of the trickster in the mythology of many cultures.

[...]

Manin is a bird whose vision extends far beyond the territory of mathematics into the wider landscape of human culture. One of his hobbies is the theory of archetypes invented by the Swiss psychologist Carl Jung. An archetype, according to Jung, is a mental image rooted in a collective unconscious that we all share. The intense emotions that archetypes carry with them are relics of lost memories of collective joy and suffering. Manin is saying that we do not need to accept Jung's theory as true in order to find it illuminating.

2. In The Trouble With Physics Lee Smolin writes of "Seers" who have something in common with Dyson's "Birds":

Seers are very different. They are dreamers. They go to into science because they have questions about the nature of existence that their schoolbooks don't answer. If they weren't scientists, they might be artists or writers or they might end up in divinity school.

[...]

A common complaint of the seers is that the standard education in physics ignores the historical and philosophical context in which science develops. As Einstein said in a letter to a young physicist who had been thwarted in his attempts to add philosophy to his physics courses:

"I fully agree with you about the significance and educational value of methodology as well as history and philosophy of science. So many people today - and even professional scientists - seem to me like someone who has seen thousands of trees but has never seen a forest. A knowledge of the historical and philosophical background gives that kind of independence from prejudices of his generation from which most scientists are suffering."

3. Thomas remarks that Yuri Manin has written about how "mathematics chooses us" and "emotional platonism" which are characteristic of shamanism and that the number theorist Kazuya Kato writes "Mysterious properties of zeta values seem to tell us (in a not so loud voice) that our universe has the same properties: The universe is not explained just by real numbers. It has p-adic properties … We ourselves may have the same properties" which fits into the shamanistic way of thinking ("knowing something is becoming it").

4. In his autobiography titled The Apprentice of a Mathematician, Andre Weil wrote about how he was heavily influenced by Hindu thought and studied Sanskrit and mystic Hindu poetry.

5. According to Allyn Jackson's article on Alexander Grothendieck:

Honig once asked Grothendieck why he had gone into mathematics. Grothendieck replied that he had two special passions, mathematics and piano, but he chose mathematics because he thought it would be easier to earn a living that way. His gift for mathematics was so abundantly clear, said Honig, “I was astonished that at any moment he could hesitate between mathematics and music.”

Grothendieck's interest in music is corroborated by Luc Illusie who said:

Grothendieck had a very strong feeling for music. He liked Bach and his most beloved pieceswere the last quartets by Beethoven.

According to Winifred Scharlau's Who is Alexander Grothendieck?

From 1974 Grothendieck turned to Buddhism; several times he was visited by Japanese monks from the order Nipponzan Myohoji (in English the name translates roughly as “Japanese community of the wonderful lotus sutra”), which preaches strict nonviolence and erects peace pagodas throughout the world. But his attachment to Buddhism did not last. From around 1980 Grothendieck gravitated toward Christian mystical and esoteric ideas.

and

It appears that he thoroughly worked through, for example, Freud's Traumdeutung (The Interpretation of Dreams) and also read other relevant literature.

6. According to Frank Wilczek's Introduction to Philosophy of Mathematics and Natural Sciences by Hermann Weyl,

In his preface Weyl says, "I was also bound, though less consciously, by the German literary and philosophical tradition in which I had grown up" (xv). It was in fact a cosmopolitan tradition, of which *Philosophy of Mathematics and Natural Science* might be the last great expression. Decartes, Leibniz, Hume, and Kant are taken as familiar friends and interlocutors. Weyl's eruidition is, implicitly, a touching affirmation of a community of mind and inquiry stretching across time and space, and progressing through experience, reflection, and open dialogue.

7. In Robert Langlands' Lectures on the Practice of Mathematics and Is There Beauty in Mathematical Theories?, Langlands discusses the history of mathematics at length and quotes Rainer Maria Rilke, and Rudyard Kipling.

8. Some examples of famous birds who identify as geometers in a broad sense are Bernhard Riemann, Henri Poincare, Felix Klein, Elie Cartan, Andre Weil, Shiing-Shen Chern, Alexander Grothendieck, Raoul Bott, Friedrich Hirzebruch, Michael Atiyah, Yuri Manin, Barry Mazur, Alain Connes, Bill Thurston, Mikhail Gromov.

Correlates of the frog category

My impression is that frogs tend to be highly detail-oriented, conservative (in the sense that Rota describes), have a good memory of lots of facts, high technical prowess, ability to focus on a problem a very long time, and be interested in areas of math like elementary and analytic number theory, analysis, group theory and combinatorics. Here I'll give some supporting evidence. I believe that the thinkers discussed would identify themselves as frogs.

1. The conservative quality of frogs that Rota alludes to is negatively correlated with openness to experience. For an example of a conservative frog, I would cite Harold Davenport:

Davenport was a natural conservative. "All change is for the worse" he used to say with complete conviction. He was entirely out of sympathy with the waves of change in the teaching of mathematics but accepted them as an inevitable evil. Selective in his enjoyment of modern technology, he never entered an aeroplane, would use a lift if no alternative existed (at the International Congress in Moscow he trudged up and down the interminable stairs of Stalin's skyscraper), and preferred to send his papers for publication written in his characteristically neat hand.

Davenport's frog aesthetic comes across in his remark

Great mathematics is achieved by solving difficult problems not by fabricating elaborate theories in search of a problem.

2. The Odd Order Theorem in finite group theory is a seminal result which was proved by the two frogs Walter Feit and John Thompson. One of my friends who did his PhD in finite group theory said that understanding *a single line* of their 250 page proof requires a serious effort. In a 1985 interview, Jean-Pierre Serre said

Chevalley once tried to take this as the topic of a seminar, with the idea of giving a complete account of the proof. After two years, he had to give up.

Claude Chevalley was an outstandingly good mathematician. I read the fact that somebody of such high caliber had much trouble as he did with the proof as an indication of Feit and Thompson having unusually high technical prowess and ability to focus on a single problem for a long time even *relative to other remarkable mathematicians*. This is counterbalanced by the mathematical output of Feit and Thompson was essentially restricted to the topic of finite groups theory in contrast with that of many mathematicians who have broader interests.

3. The identification of combinatorics as a mathematical field populated by problem solvers comes across in Gowers' essay linked above. The subject of elementary number theory was very heavily influenced by Erdos who has been labeled a canonical problem solver. In the introduction to a course in analytic number theory, Noam Elkies wrote

It has often been said that there are two kinds of mathematicians: theory builders and problem solvers. In the mathematics of our century these two styles are epitomized respectively by A. Grothendieck and P. Erdos [...] analytic number theory as usually practiced falls in the problem-solving camp.

Two of the founders of the mathematical field of analysis, namely Cauchy and Weierstrass were frogs. Klein writes

...in the twenties Cauchy (1789-1857) developed, in lectures and in books, the first rigorous foundings of infinitesimal calculus in the modern sense. He not only gives an exact definition of the differential quotient, and of the integral, by means of the limit of a finite quotient and of a finite sum, respectively, as had previously been done, at times; but, by means of the mean-value theorem he erects upon this, for the first time, a consistent structure for the infinitesimal calculus [...] These theories also partake of the nature of plan A, since they work overthe field in a logical systematic way, quite apart from other branches of knowledge.

[...]

From the middle of the century on, the method of thought A comes again to the front with Weierstrass (1815-1897) [...] I have already investigated Weierstrass function theory as an example of A.

Many of the prominent contemporary analysts like John Nash and Grigori Perelman are problem solvers.

Correlates of the beaver category

My impression is that beavers tend to be interested in jigsaw puzzles, word puzzles, logic puzzles, board games like Go, sorting tasks, algorithms, computational complexity, logic, respond best to a stream of immediate feedback in the way of tangible progress and can have trouble focusing on learning mathematical subjects in ways that require a lot of development before one engages in computation. Many computer scientists seem to me to fall into the beaver paradigm. Here I'm on shakier ground as I've seen little public discussion of beavers and most of what I've observed that supports my impression is born of subjective experience with people who I know, but I'll try to give some examples that seem to me to fall into the beaver paradigm:

1. Doron Zeilberger's focus on algorithmics, ultrafinitism and constructivist mathematics.

2. Harold Edwards' focus on constructivist mathematics (which comes across in his books titled *Higher Arithmetic: An Algorithmic Introduction to Number Theory*, *Essays in Constructive Mathematics, Galois Theory, Fermat's Last Theorem* and *Divisor Theory*) is in the beaver paradigm.

3. Terence Tao's interest in logic puzzles like the blue-eyed islanders puzzle and his interest in ultrafilters and nonstandard analysis.

4. The work of Jonathan Borwein and Peter Borwein computing a billion digits of pi.

5. The computational work of historically great mathematicians like Newton Euler, Gauss, Jacobi, and Ramanujan.

6. Don Zagier's remark in his essay in Mariana Cook's book

Even now I am not a modern mathematician and very abstract ideas are unnatural for me. I have of course learned to work with them but I haven't really internalized it and remain a concrete mathematician. I like explicit, hands-on formulas. To me they have a beauty of their own. They can be deep or not.

7. The focus on explicit formulae in the area of q-series.

8. Interest in Newcomb's Problem and its variations.

9. Scott Aaronson's interest in computational complexity, algorithms, and questions between logic and algorithmics as reflected in his MathOverflow post Succinctly naming big numbers: ZFC versus Busy-Beaver.

To Be Continued

In a future post I will describe superficial similarities and superficial differences between the three types and misunderstandings between different types which arise from generalizing from one example and cultural differences. Regarding his mastercraftspeople/seers dicotomy, Lee Smolin says

It is only to be expected that members of these two groups misunderstand and mistrust each other.

Timothy Gowers writes about how there's a schism between his two categories of mathematicians and says

this is not an entirely healthy state of a ffairs.

As Dyson and Zeilberger said, all three types are important to scientific progress. I believe that intellectual progress will be increase if the three types can learn to better understand each other.