Παραδείγματα Υπολογισμού Ορισμένου Ολοκληρώματος

Κλασσικό Παράδειγμα

```
> restart :
>
> x := i→a + (b − a)/n · i;
```

$$x := i \to a + \frac{(b-a)\,i}{n} \tag{1.1.1}$$

```
>
> ξ := i→1/2 · (x(i − 1) + x(i));
```

$$\xi := i \to \frac{1}{2}\,x(i-1) + \frac{1}{2}\,x(i) \tag{1.1.2}$$

```
>
> f := x→x^3;
```

$$f := x \to x^3 \tag{1.1.3}$$

```
>
> simplify(f(ξ(i)));
```

$$-\frac{1}{8}\,\frac{(-2\,a\,n - 2\,b\,i + b + 2\,a\,i - a)^3}{n^3} \tag{1.1.4}$$

```
>
> lim[n→∞](sum((b − a)/n · simplify(f(ξ(i))), i = 1..n));
```

$$\frac{1}{4}\,b^4 - \frac{1}{4}\,a^4 \tag{1.1.5}$$

```
> int(f(x), x = a..b, 'AllSolutions');
```

$$\frac{1}{4}b^4 - \frac{1}{4}a^4 \qquad \textbf{(1.1.6)}$$

```
>
```

Εφαρμογή σελ 337

```
> restart:
>
> x := i->a + (b - a)/n*i;
```

$$x := i \to a + \frac{(b-a)\,i}{n} \qquad \textbf{(1.2.1)}$$

```
>
> ξ := i->1/2*(x(i - 1) + x(i));
```

$$\xi := i \to \frac{1}{2}x(i-1) + \frac{1}{2}x(i) \qquad \textbf{(1.2.2)}$$

```
>
> f := x->(x + 1)/sqrt(x);
```

$$f := x \to \frac{x+1}{\sqrt{x}} \qquad \textbf{(1.2.3)}$$

```
>
> simplify(f(ξ(i)));
```

$$-\frac{-2an - 2bi + b + 2ai - a - 2n}{n\sqrt{-\dfrac{-4an - 4bi + 2b + 4ai - 2a}{n}}} \qquad \textbf{(1.2.4)}$$

```
> 
> limit(sum((b-a)/n*simplify(f(xi(i))),i=1..n),n=infinity);
```

$$-\frac{2}{3}a^{3/2}-2\sqrt{a}+2\sqrt{b}+\frac{2}{3}b^{3/2} \tag{1.2.5}$$

```
> 
> int(f(x),x=a..b);
```

$$-\frac{2}{3}a^{3/2}-2\sqrt{a}+2\sqrt{b}+\frac{2}{3}b^{3/2} \tag{1.2.6}$$

```
> 
> a := 1: b := 4:
> 
> limit(sum((b-a)/n*simplify(f(xi(i))),i=1..n),n=infinity);
```

$$\frac{20}{3} \tag{1.2.7}$$

```
> 
> int(f(x),x=a..b,'AllSolutions');
```

$$\frac{20}{3} \tag{1.2.8}$$

```
> 
```

Ασκηση Πανελλήνιες 2003

```
> restart:
>
> x := i->a+(b-a)/n*i;
```

$$x := i \to a + \frac{(b-a)\,i}{n} \qquad (1.3.1)$$

```
>
> ξ := i->1/2*(x(i-1)+x(i));
```

$$\xi := i \to \frac{1}{2}x(i-1) + \frac{1}{2}x(i) \qquad (1.3.2)$$

```
>
> a := 0: b := 1:
>
> f := x->1/sqrt(1+x^2);
```

$$f := x \to \frac{1}{\sqrt{1+x^2}} \qquad (1.3.3)$$

```
>
> simplify(f(ξ(i)));
```

$$\frac{2}{\sqrt{\dfrac{4n^2+4i^2-4i+1}{n^2}}} \qquad (1.3.4)$$

```
>
```

```
> limit(sum((b-a)/n*simplify(f(ξ(i))),i=1..n),n=infinity);
```

$$\ln\left(1+\sqrt{2}\right) \tag{1.3.5}$$

```
> int(f(x),x=a..b,'AllSolutions');
```

$$-\ln\left(\sqrt{2}-1\right) \tag{1.3.6}$$

Ασκηση 2 σελίδα 352

```
> restart:
> x:=i->a+(b-a)/n*i;
```

$$x := i \to a + \frac{(b-a)\,i}{n} \tag{1.4.1}$$

```
> ξ:=i->1/2*(x(i-1)+x(i));
```

$$\xi := i \to \frac{1}{2}x(i-1) + \frac{1}{2}x(i) \tag{1.4.2}$$

```
> f:=x->1/sin(x);
```

$$f := x \to \frac{1}{\sin(x)} \tag{1.4.3}$$

```
> simplify(f(ξ(i)));
```

$$-\frac{1}{\sin\left(\frac{1}{2}\,\frac{-2an-2bi+b+2ai-a}{n}\right)} \tag{1.4.4}$$

```
> a := π/3 : b := π/2 :
> limit(sum((b-a)/n * simplify(f(ξ(i))), i=1..n), n=infinity);
```

$$-\ln\left(\frac{1}{3}\sqrt{3}\right) \tag{1.4.5}$$

```
> int(f(x), x = a..b, 'AllSolutions');
```

$$\frac{1}{2}\ln(3) \tag{1.4.6}$$

integrale impropre

```
> restart :
> x := i -> a + (b-a)/n * i;
```

$$x := i \to a + \frac{(b-a)\,i}{n} \tag{1.5.1}$$

```
> ξ := i→ 1/2 · (x(i − 1) + x(i));
```

$$\xi := i \to \frac{1}{2}\, x(i-1) + \frac{1}{2}\, x(i) \tag{1.5.2}$$

```
>
> f := x→ 4/sqrt(4 − x^2);
```

$$f := x \to \frac{4}{\sqrt{4 - x^2}} \tag{1.5.3}$$

```
>
> simplify(f(ξ(i)));
```

$$\frac{8}{\sqrt{-\dfrac{-16n^2 + 8bia - 8bi^2a - 4anb + 8anbi - 8a^2ni + 4a^2n^2 + 4a^2n + 4b^2i^2 - 4b^2i + b^2 - 2ba + 4a^2i^2 - 4a^2i + a^2}{n^2}}} \tag{1.5.4}$$

```
>
> a := 0 : b := 2 :
>
> lim[n→∞] ( Σ_{i=1}^{n} (b − a)/n · simplify(f(ξ(i))) );
```

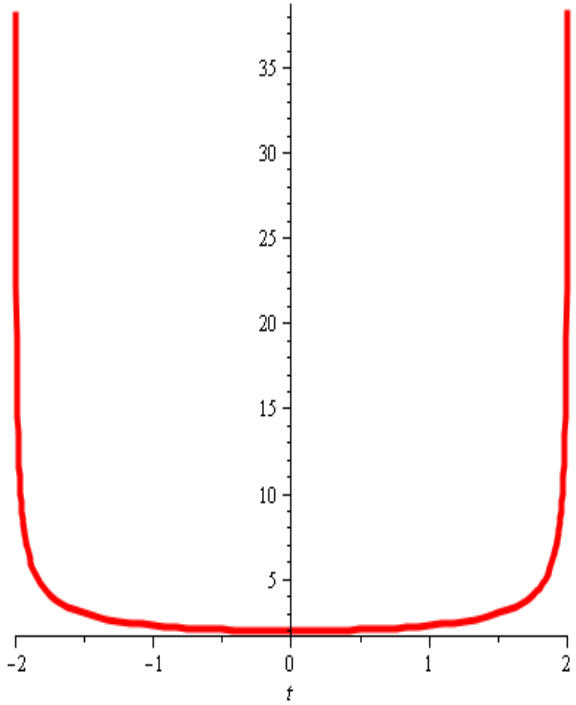
$$2\pi \tag{1.5.5}$$

```
>
> int(f(x), x = a..b, 'AllSolutions');
```

$$2\pi \tag{1.5.6}$$

```
>
> with(plots) :
>
```

```
> plot(4/sqrt(4 - t^2), t = -2..2);
```




```
>
```

Η συνάρτηση erf:=2/sqrt(π) * int(exp(-t^2), t = 0..x)

```
> restart :
```

```
> x := i->a + (b-a)/n*i;
```

$$x := i \to a + \frac{(b-a)\,i}{n} \tag{2.1}$$

```
> 
> ξ := i->1/2*(x(i-1)+x(i));
```

$$\xi := i \to \frac{1}{2}x(i-1) + \frac{1}{2}x(i) \tag{2.2}$$

```
> 
> f := x->2/sqrt(π)*int(exp(-t^2),t=0..x);
```

$$f := x \to \frac{2\left(\int_0^x e^{-t^2}\,dt\right)}{\sqrt{\pi}} \tag{2.3}$$

```
> 
> simplify(f(ξ(i)));
```

$$-\operatorname{erf}\left(\frac{1}{2}\,\frac{-2an+2ai-a-2bi+b}{n}\right) \tag{2.4}$$

```
> 
> limit(sum((b-a)/n*simplify(f(ξ(i))),i=1..n),n=infinity);
```

$$-\frac{a\operatorname{erf}(a)\sqrt{\pi} + e^{-a^2} - b\operatorname{erf}(b)\sqrt{\pi} - e^{-b^2}}{\sqrt{\pi}} \tag{2.5}$$

```
> 
```

```
> int(f(x), x = a..b);
```

$$-\frac{a\,\mathrm{erf}(a)\sqrt{\pi} + e^{-a^2} - b\,\mathrm{erf}(b)\sqrt{\pi} - e^{-b^2}}{\sqrt{\pi}} \qquad (2.6)$$

```
>
```

Παράγωγος 1

```
> restart :
>
> f := x→ 4/sqrt(4 - x^2);
```

$$f := x \to \frac{4}{\sqrt{4 - x^2}} \qquad (3.1)$$

```
>
> diffq := (x, h)→ (f(x + h) - f(x))/h;
```

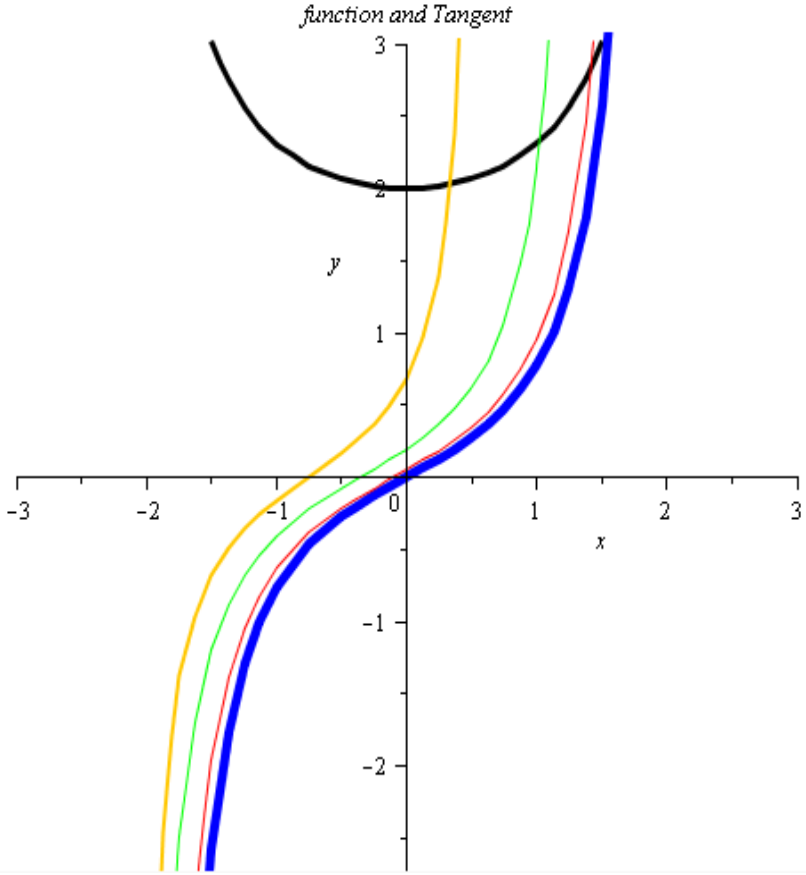
$$diffq := (x, h) \to \frac{f(x + h) - f(x)}{h} \qquad (3.2)$$

```
>
>
> lim[h → 0] diffq(x, h);
```

$$\frac{4x}{(4 - x^2)^{3/2}} \qquad (3.3)$$

```
>
> with(plots) :
```

```
> Q := plot(f(x), x = -3 ..3, y = -3 ..3, color = black) :
> P := plot(diff(f(x), x), x = -3 ..3, y = -3 ..3, style = patch) :
> G := plot(diffq(x, 0.2), x = -3 ..3, y = -3 ..3, style = point, color = blue) :
> F := plot(diffq(x, 0.7), x = -3 ..3, y = -3 ..3, style = point, color = red) :
> H := plot(diffq(x, 1.5), x = -3 ..3, y = -3 ..3, style = point, color = green) :
> display({Q, P, G, F, H}, scaling = constrained, title = `function and Tangent`);
```



```
> f := x→erf(x);
```

$$f := x \to \mathrm{erf}(x) \tag{3.4}$$

```
>
> diffq(x, h);
```

$$\frac{\mathrm{erf}(x+h) - \mathrm{erf}(x)}{h} \tag{3.5}$$

```
> limit(diffq(x, h), h = 0);
```

$$\frac{2}{\sqrt{\pi}\, e^{x^2}} \tag{3.6}$$

```
>
```

Παράγωγος 2

```
> restart :
>
> f := x→(3·x^2 - 3·x + 1)·cos(x^3 - 3·x^2 + 4·x - 2);
```

$$f := x \to \frac{4}{\sqrt{4 - x^2}} \tag{4.1}$$

```
>
> diffq := (x, h)→(f(x + h) - f(x))/h;
```

$$diffq := (x, h) \to \frac{f(x+h) - f(x)}{h} \tag{4.2}$$

```
>
>
```

```
> 
> lim[h → 0] diffq(x, h);
```

$$\frac{4x}{(4-x^2)^{3/2}} \qquad (4.3)$$

```
> 
```