

έκρινα ορθό να σου εκθέσω
μία μέθοδο η οποία να σου
επιτρέπει να λαμβάνεις
αφορμές ώστε να μπορείς να
εξετάζεις μερικές μαθηματικές
προτάσεις με Μηχανική [...] *...*
διότι είναι πιο εύκολο, αφού
κάνεις βρει με αυτό το τρόπο
(της Μηχανικής) κάποια
γνώση, να βρει μετά την
(Γεωμετρική) απόδειξη αυτή
να ερευνά χωρίς να
γνωρίζει τίποτα. [...] είμαι
πεπεισμένος ότι προσφέρω
όχι μικρή υπηρεσία στα
Μαθηματικά.

Αρχιμήδης, Έφοδος



Benjamin West (1738 – 1820)

Να βρεθεί η τιμή του

$$\frac{\sum_{k=0}^{\infty} \frac{(2k)!}{(k!)^3}}{\sum_{k=0}^{\infty} \frac{1}{(k!)^2}}$$

άθροισμα 1000 όρων του αριθμητή = 16.84398368

άθροισμα 1000 όρων του παρονομαστή = 2.279585302

αριθμητής δια παρονομαστή = 7.389056099

δηλαδή πόσο;

$$\sqrt{7,389056099} = 2,718281828$$
$$= e$$

Δηλαδή αναμένουμε

$$\frac{\sum_{k=0}^{\infty} \frac{(2k)!}{(k!)^3}}{\sum_{k=0}^{\infty} \frac{1}{(k!)^2}} = e^2$$

(τελικά αποδεικνύεται ότι $\sum_{k=0}^{\infty} \frac{(2k)!}{(k!)^3} x^k = e^{2x} \sum_{k=0}^{\infty} \frac{1}{(k!)^2} x^k$)

INVERSE SYMBOLIC CALCULATOR

Please enter a number or a Maple expression:

- Simple Lookup and Browser** for any number.
- Smart Lookup** for any number.
- Generalized Expansions** for real numbers of at least 16 digits.
- Integer Relation Algorithms** for any number.



Expressions that are **not** numeric like $\ln(\text{Pi}*\text{sqrt}(2))$ are evaluated in [Maple](#) in symbolic form first, followed by a floating point evaluation followed by a lookup.

7389054793600042 = (0187) $\text{Prod}(1-1/(2^n * (2/3 * n^3 - 3 * n^2 + 76/3 * n - 21)), n=1..inf)$

7389054949332862 = (0192) $(-\arctan(1/2) + 2/3) / (-\text{Madelung} + 1)$

7389055042942944 = (0015) $\text{sum}((5/6 * n^3 - n^2 + 85/6 * n + 5) * n! / n^n, n=1..inf)$

7389055275601539 = (0397) $\text{sum}(\text{A008862}(n) / (n-1)!, \text{ from E.I.S.})$

7389055435069957 = (0001) $\text{GAM}(1/12) ^ \ln(5) * 2 ^ (1/3) ^ \ln(5)$

7389055673250783 = (0001) $\text{BesK}(1, 1) * (\text{Gibbs-Golomb})$

7389056024687140 = (0397) $\text{sum}(\text{A008863}(n) / (n-1)!, \text{ from E.I.S.})$

7389056074776242 = (0333) $1/4 * (14 - 4 * 3 ^ (1/4)) ^ (1/2)$

7389056098930646 = (0398) $\text{sum}(\text{A001787}(n) / n!, \text{ from E.I.S.})$

7389056098930650 = (0000) $\exp(2)$

7389056098930650 = (0001) $E * \exp(1/\text{Pi}) ^ \text{Pi}$

7389056098930650 = (0001) E^2

7389056098930650 = (0001) $\exp(1/\text{Pi}) ^ (\text{GAM}(1/6) * \text{GAM}(5/6))$

7389056098930650 = (0026) $e ^ (2^1) \text{ normalized}$

7389056098930650 = (0276) $\exp(\text{roots}(-2 * x^2 + 2 * x + 4))$

7389056098930650 = (0397) $\text{sum}(\text{A000079}(n) / (n-1)!, \text{ from E.I.S.})$

7389056098930650 = (0405) $\exp(2)$

7389056442785247 = (0264) $\text{sum}(1 / (-2 * 4^n + 5^n + 2 * 9^n), n=1..inf)$

7389056527781341 = (0191) $\text{Prod}(1 - (23/6 * n^3 - 21 * n^2 + 241/6 * n - 9) / 2^n, n=1..inf)$

7389056702761454 = (0180) $\text{sum}(1 / (23/6 * n^3 - 33/2 * n^2 + 92/3 * n + 3) + C(3 * n, n), n=1..inf)$

7389056864967827 = (0001) $(\ln(2) * \text{NineConst} - \ln(2 + \text{sr}(3))) / \ln(2)$

7389058766366095 = (0187) $\text{Prod}(1 - 1 / (2^n * (1/3 * n^3 + 5/2 * n^2 + 61/6 * n - 11)), n=1..inf)$

7389059254662929 = (0258) $F(9/11, 2/11; 1/9, 3/7; 1)$

7389060887512899 = (0399) $716/969$

7389061600005478 = (0190) $\text{Prod}(1 - 1 / (C(2 * n, n) * (11/6 * n^3 - 17/2 * n^2 + 71/3 * n - 15)), n=1..inf)$

7389062264527210 = (0263) $13 + 13 * x - 4 * x^2 + 3 * x^3$

7389062921484646 = (0003) $\text{sum}(1 / (5^n + (8/3 * n^3 - 19/2 * n^2 + 173/6 * n - 7)), n=1..inf)$

7389062983879452 = (0001) $(\text{BesK}(1, 1) - \text{Zeta}(3)) ^ \text{Lehmer}$

7389063871850955 = (0400) $\text{sum}(1 / (n! + 1/2 * n^2 + 5/2 * n - 2), n=1..inf)$

7389063898680500 = (0261) $-5 + 3 * x + 6 * x^3 - x^4 + 3 * x^5$

7389064071872131 = (0131) $\text{sum}(1 / (40 * n^2 - 108 * n + 99), n=1..inf)$

7389064476564540 = (0188) $\text{Prod}(1 - 1 / (7/6 * n^3 - 4 * n^2 + 143/6 * n - 18 + 2^n), n=1..inf)$

7389064553791850 = (0002) $\text{sum}(1 / (3^n * (53/2 * n^2 - 111/2 * n + 87)), n=1..inf)$

7389064677490938 = (0315) $-3 + x + 5 * x^2 + x^4 + 8 * x^8$

7389064774326998 = (0003) $\text{sum}(1 / (3^n + (5 * n^2 - 13 * n + 42)), n=1..inf)$

7389065528711479 = (0333) $13 ^ (1/2) * 2 ^ (1/3) * 7 ^ (1/4)$

7389066020639091 = (0015) $\text{sum}((13/6 * n^3 - n^2 - 25/6 * n + 17) * n! / n^n, n=1..inf)$

7389066074630174 = (0131) $\text{sum}(1 / (38 * n^2 - 71 * n + 56), n=1..inf)$

Done

Inverse Symbolic Calculator:

<http://oldweb.cecm.sfu.ca/projects/ISC/ISCmain.html>

$$e^2 = 7,389056098$$

$$\frac{716}{969} = 0,7389060888$$

$$\frac{1}{4}\sqrt{14 - 4\sqrt[4]{3}} = 0,7389056075$$

ρίζα της $3x^3 - 4x^2 + 13x + 13 = 0$ είναι $-0,7389062264527210$

$$\sum_{k=1}^{\infty} \frac{1}{-2 \cdot 4^k + 5^k - 2 \cdot 9^k} = 0,07389056442$$

αν τον ήξερε ο Euler θα διαπίστωνε ότι $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$

$$x^{25} - 1 = (x-1)(x^4+x^3+x^2+x+1)(x^{20}+x^{15}+x^{10}+x^5+1)$$

$$x^{26} - 1 = (x-1)(x+1)$$

$$(x^{12}+x^{11}+x^{10}+x^9+x^8+x^7+x^6+x^5+x^4+x^3+x^2+x+1)$$

$$(x^{12}-x^{11}+x^{10}-x^9+x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1)$$

$$x^{27} - 1 = (x-1)(x^2+x+1)$$

$$(x^6+x^3+1)(x^{18}+x^9+1)$$

$$x^{100} - 1 = (x-1)(x+1)(x^2+1)(x^4+x^3+x^2+x+1)$$

$$(x^4-x^3+x^2-x+1)(x^8-x^6+x^4-x^2+1)$$

$$(x^{20}+x^{15}+x^{10}+x^5+1)(x^{20}-x^{15}+x^{10}-x^5+1)$$

$$(x^{40}-x^{30}+x^{20}-x^{10}+1)$$

$$\begin{aligned}
x^{105} - 1 = & (x-1)(x^2+x+1)(x^4+x^3+x^2+x+1) \\
& (x^6+x^5+x^4+x^3+x^2+x+1) \\
& (x^8-x^7+x^5-x^4+x^3-x+1) \\
& (x^{12}-x^{11}+x^9-x^8+x^7-x^6+x^5-x+1) \\
& (x^{24}-x^{23}+x^{19}-x^{18}+x^{17}-x^{16}+x^{14}-x^{13}+x^{12}-x^{11}+ \\
& \quad + x^{10}-x^8+x^{14}-x+1) \\
& (x^{48}+x^{47}+x^{46}-x^{43}-x^{42}-2x^{41}-x^{40}-x^{39}+x^{36}+ \\
& \quad + x^{35}+x^{34}+x^{33}+x^{32}+x^{31}-x^{28}-x^{26}-x^{24}+ \\
& \quad -x^{22}-x^{20}+x^{17}+x^{16}+x^{15}+x^{14}+x^{13}+x^{12} \\
& \quad -x^9-x^8-2x^7-x^6-x^5+x^2+x+1)
\end{aligned}$$

$$\int 2x(x^2 + 1)^{100} dx = \frac{1}{101} (x^2 + 1)^{101} + c$$

$$\begin{aligned} & \frac{1}{101} x^{202} + x^{200} + 50 x^{198} + 1650 x^{196} + 40425 x^{194} + 784245 x^{192} + 12547920 x^{190} \\ & + 170293200 x^{188} + 2000945100 x^{186} + 20676432700 x^{184} + 190223180840 x^{182} \\ & + 1573664496040 x^{180} + 11802483720300 x^{178} + 80801619315900 x^{176} \\ & + 507895892842800 x^{174} + 2945796178488240 x^{172} + 15833654459374290 x^{170} \\ & + 79168272296871450 x^{168} + 369451937385400100 x^{166} \\ & + 1613921621209905700 x^{164} + 6617078646960613370 x^{162} \\ & + 25523017638276651570 x^{160} + 92810973230096914800 x^{158} \\ & + 318785516746854620400 x^{156} + 1036052929427277516300 x^{154} \\ & + 3191043022636014750204 x^{152} + 9327664220012966192904 x^{150} \\ & + 25910178388924906091400 x^{148} + 68476900027872966098700 x^{146} \\ & + 172372886277059535351900 x^{144} + 413694927064942884844560 x^{142} \\ & + 947494832955191768514960 x^{140} + 2072644947089481993626475 x^{138} \\ & + 4333712162096189623037175 x^{136} + 8667424324192379246074350 x^{134} \\ & + 16591926563453983128199470 x^{132} + 30418532032998969068365695 x^{130} \\ & + 53437961679592783498480275 x^{128} + 90000777565629951155335200 x^{126} \end{aligned}$$

$$\begin{aligned} &+ 145385871452171459558618400 x^{124} + 225348100750865762315858520 x^{122} \\ &+ 335274003556166134177252920 x^{120} + 478962862223094477396075600 x^{118} \\ &+ 657181601654943585264382800 x^{116} + 866284838545152907848504600 x^{114} \\ &+ 1097294128823860349941439160 x^{112} + 1335836330742090860798273760 x^{110} \\ &+ 1563212727464148879657554400 x^{108} + 1758614318397167489614748700 x^{106} \\ &+ 1902174670919385243869013900 x^{104} + 1978261657756160653623774456 x^{102} \\ &+ 1978261657756160653623774456 x^{100} + 1902174670919385243869013900 x^{98} \\ &+ 1758614318397167489614748700 x^{96} + 1563212727464148879657554400 x^{94} \\ &+ 1335836330742090860798273760 x^{92} + 1097294128823860349941439160 x^{90} \\ &+ 866284838545152907848504600 x^{88} + 657181601654943585264382800 x^{86} \\ &+ 478962862223094477396075600 x^{84} + 335274003556166134177252920 x^{82} \\ &+ 225348100750865762315858520 x^{80} + 145385871452171459558618400 x^{78} \\ &+ 90000777565629951155335200 x^{76} + 53437961679592783498480275 x^{74} \\ &+ 30418532032998969068365695 x^{72} + 16591926563453983128199470 x^{70} \\ &+ 8667424324192379246074350 x^{68} + 4333712162096189623037175 x^{66} \\ &+ 2072644947089481993626475 x^{64} + 947494832955191768514960 x^{62} \end{aligned}$$

$$\begin{aligned} &+ 413694927064942884844560 x^{60} + 172372886277059535351900 x^{58} \\ &+ 68476900027872966098700 x^{56} + 25910178388924906091400 x^{54} \\ &+ 9327664220012966192904 x^{52} + 3191043022636014750204 x^{50} \\ &+ 1036052929427277516300 x^{48} + 318785516746854620400 x^{46} \\ &+ 92810973230096914800 x^{44} + 25523017638276651570 x^{42} \\ &+ 6617078646960613370 x^{40} + 1613921621209905700 x^{38} \\ &+ 369451937385400100 x^{36} + 79168272296871450 x^{34} + 15833654459374290 x^{32} \\ &+ 2945796178488240 x^{30} + 507895892842800 x^{28} + 80801619315900 x^{26} \\ &+ 11802483720300 x^{24} + 1573664496040 x^{22} + 190223180840 x^{20} \\ &+ 20676432700 x^{18} + 2000945100 x^{16} + 170293200 x^{14} + 12547920 x^{12} \\ &+ 784245 x^{10} + 40425 x^8 + 1650 x^6 + 50 x^4 + x^2 \end{aligned}$$

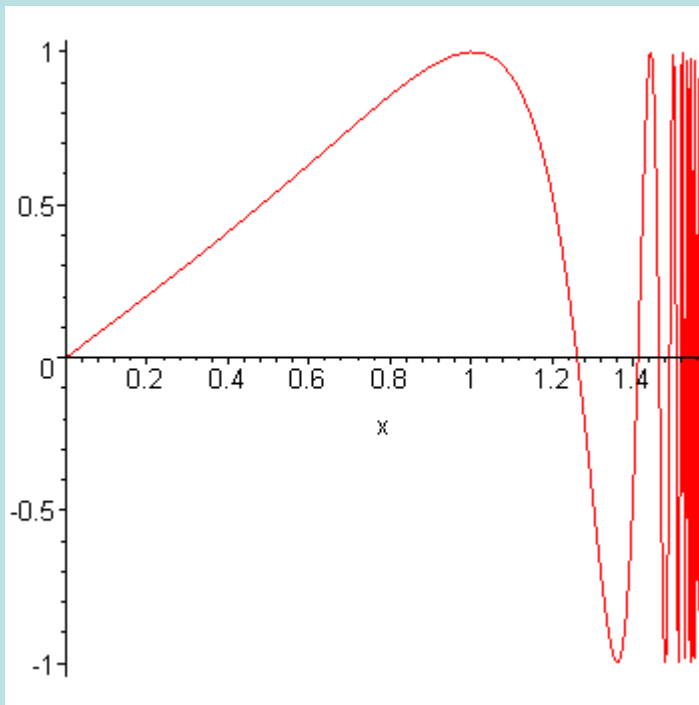
Η παραγοντοποίηση αποτυγχάνει.

Προσθέτω $\frac{1}{101}$.

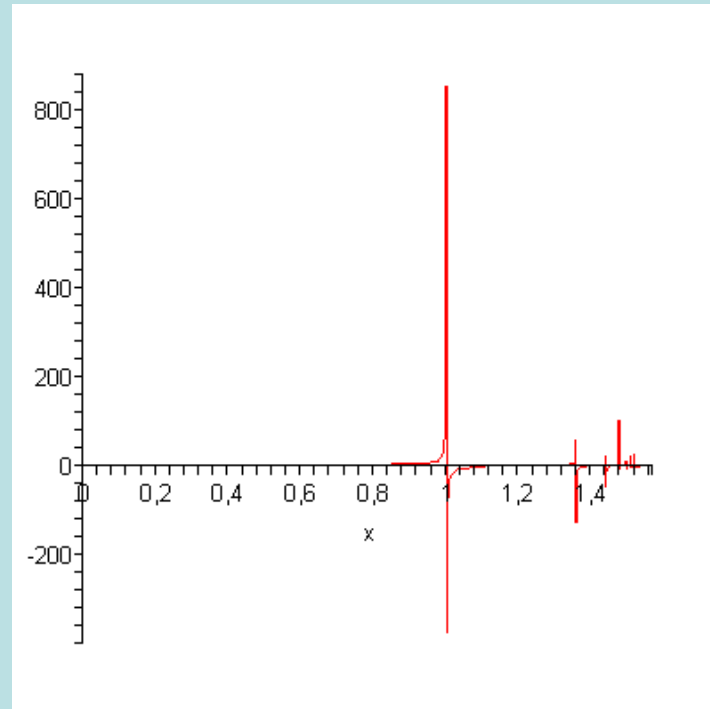
Τώρα παραγοντοποιείται. Δίνει

$$\frac{1}{101} (x^2 + 1)^{101}$$

γράφημα της $y = \eta\mu(\varepsilon\varphi(x))$

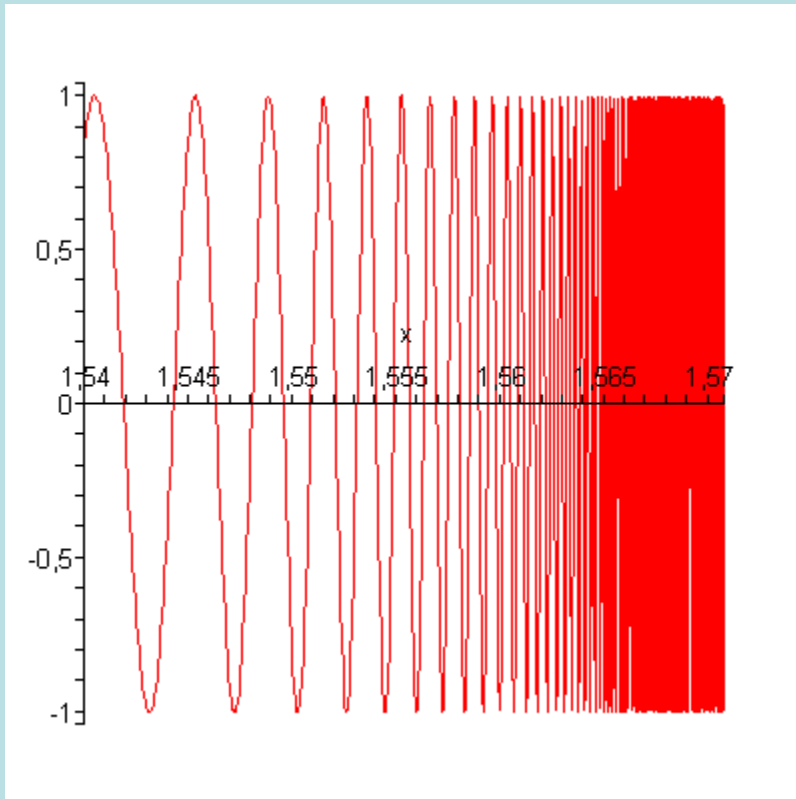


γράφημα της $y = \varepsilon\varphi(\varepsilon\varphi(x))$

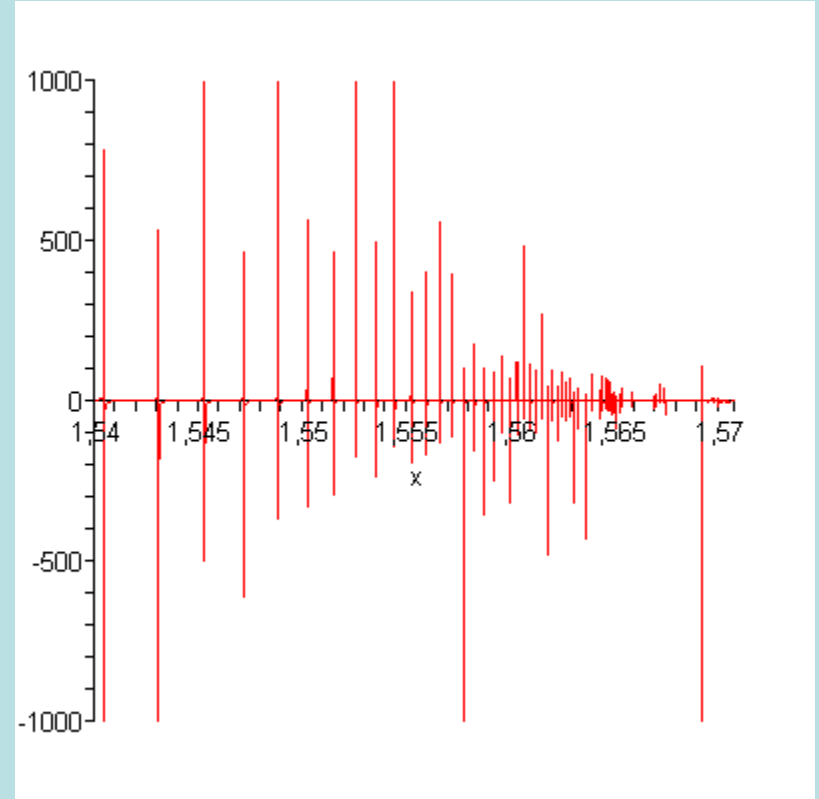


50 σημεία

γράφημα της $y = \eta\mu(\varepsilon\varphi(x))$

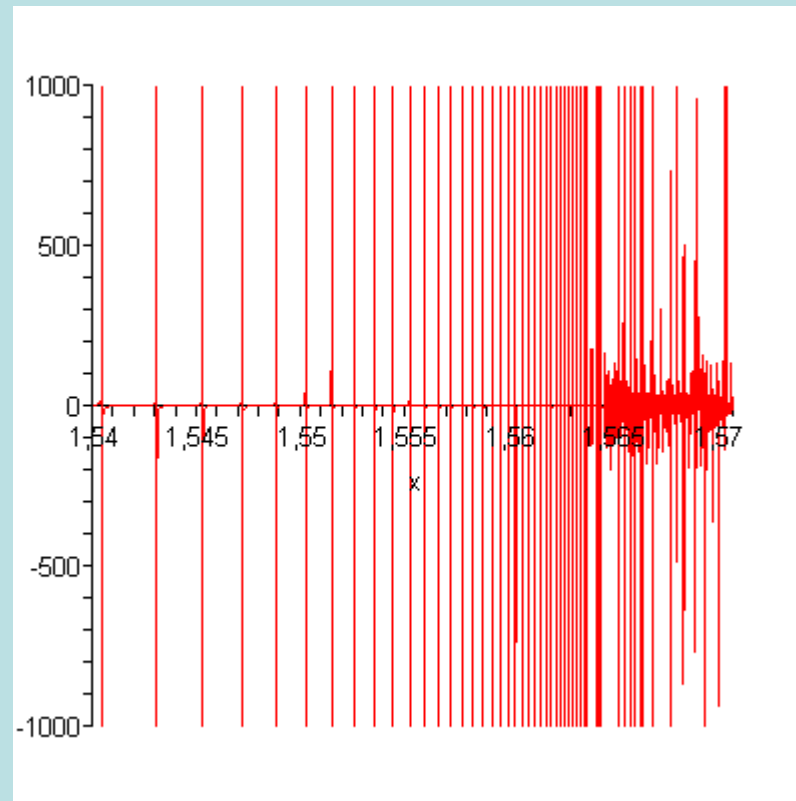


γράφημα της $y = \varepsilon\varphi(\varepsilon\varphi(x))$



500 σημεία

$$y = \varepsilon\varphi(\varepsilon\varphi(x))$$

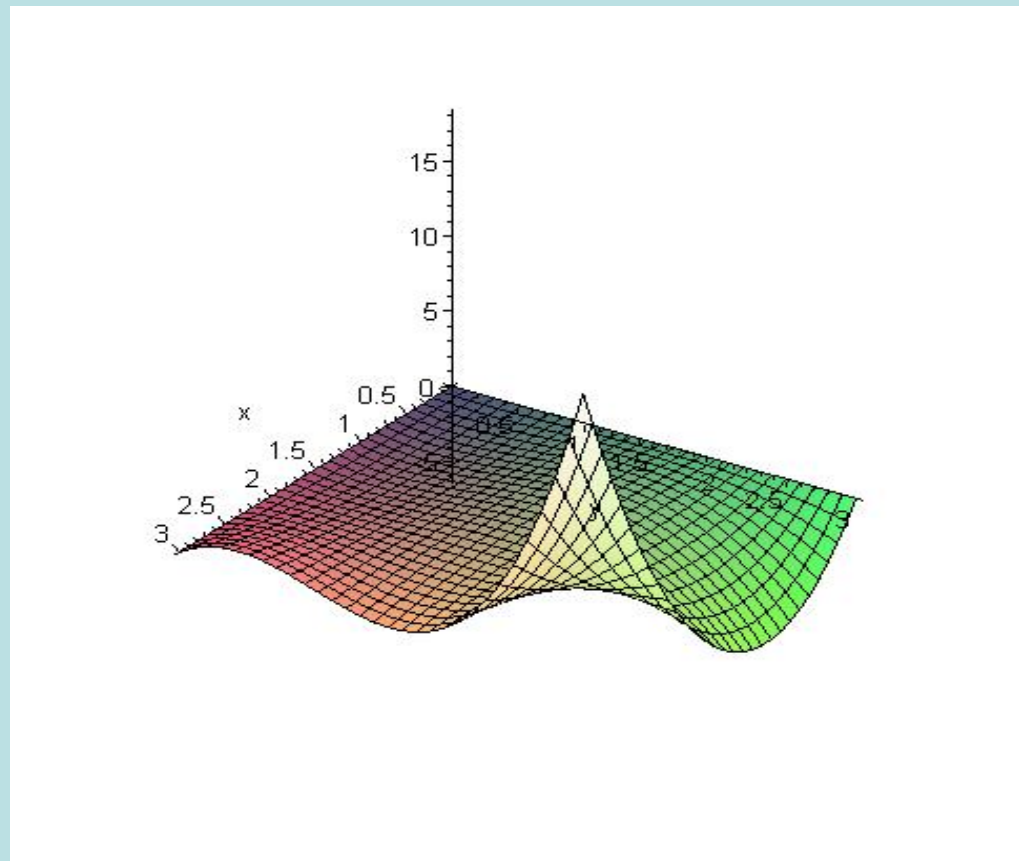
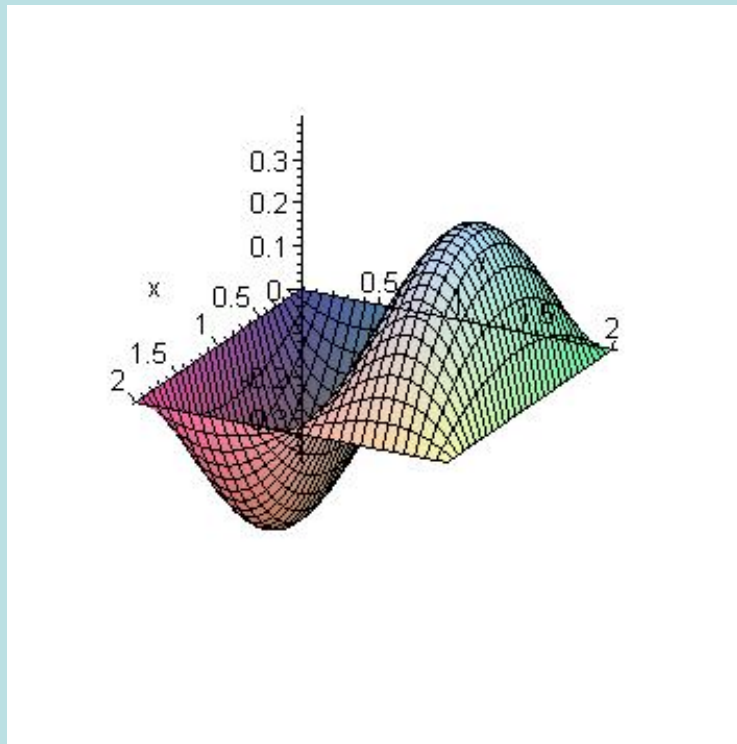


50.000 σημεία

γράφημα της $z = x(x-2)y(y-1)(y-2)$

για $0 \leq x \leq 2, 0 \leq y \leq 2$

και για $0 \leq x \leq 3, 0 \leq y \leq 3$



$$\mathbf{x_1 = \frac{1}{3}, \quad x_{n+1} = 1000x_n - 333}$$

$$\begin{aligned} \mathbf{x_1} &= \frac{1}{3}, & \mathbf{x_2} &= 1000 \cdot \frac{1}{3} - 333 \\ & & &= \frac{1000 - 999}{3} \\ & & &= \frac{1}{3} \end{aligned}$$

$$\mathbf{x_1 = x_2 = x_3 = x_4 = \dots = \frac{1}{3}}$$

$$\mathbf{x_1 = 0,33333}$$

$$\mathbf{x_2 = 1000 \cdot 0,33333 - 333 = 333,33 - 333 = 0,33}$$

$$\mathbf{x_3 = 1000 \cdot 0,33 - 333 = 330 - 333 = -3}$$

$$\mathbf{x_4 = 1000 \cdot (-3) - 333 = -3000 - 333 = -3333}$$

$$\mathbf{x_4 = 1000 \cdot (-3333) - 333 = -3333000 - 333 = -3333333}$$

$$\mathbf{x_n \rightarrow -\infty \text{ ταχύτατα}}$$