The Complex Process of Converting Tools into Mathematical Instruments: The Case of Calculators

Article *in* International Journal of Computers for Mathematical Learning · April 1998

DOI: 10.1023/A:1009892720043

2 authors, including:

Luc Trouche
Ecole normale supérieure de Lyon

LUC TROUCHE

MANAGING THE COMPLEXITY OF HUMAN/MACHINE INTERACTIONS IN COMPUTERIZED LEARNING ENVIRONMENTS: GUIDING STUDENTS' COMMAND PROCESS THROUGH INSTRUMENTAL ORCHESTRATIONS[1]

ABSTRACT. After an introduction which addresses some basic questions, this article is organized around three points: (1) The theoretical framework of the so-called "*instrumental approach*" which has been a theme in the last two CAME symposia; (2) A consideration of two processes (*instrumentalization* and *instrumentation)* which interact in the *instrumental genesis*; and (3) The introduction of the idea of instrumental *orchestration* as a way of allowing the teacher to assist the student's instrumental genesis.

KEY WORDS: artifact, mathematical instrument, instrumentation, instrumentalization, instrumental orchestration

The title of this strand of the Symposium was "Mind and Machine". A title is never neutral. It simultaneously conceals and reveals. Why not choose "Brain and Machine", "Hand and Machine", "Body and Machine", "Human and Machine", "Mind and Tool", etc.? Choosing vocabulary is always a very important question.[2] I will begin my reflection on this point: what are the main ideas shared by everybody (again the body…) about the relationships between "human" and "machines", and what are the main issues? On these issues, I will summarize my point of view and my choice of vocabulary.

1. BASIC QUESTIONS

I will introduce this article by considering first some shared ideas and then some opposing points of view related to the subject of "mind and machine".

1.1. Shared Ideas

From the outset I will use the word "tool" rather than "machine". For me, the word "machine" includes ideas of complexity and industrial manufacturing. The word "tool" is more general: a hammer is a tool, a compass is a tool and a calculator is a tool. I will use

International Journal of Computers for Mathematical Learning **9:** 281–307, 2004.

the word “tool” in the sense of something which is available for sustaining human activity. Tools can be material or cultural (such as languages, for example). When speaking of a tool before considering its users and its uses, I will speak of an *artifact* (see Section 2.1).

- The first shared idea is the importance of tools to define humankind. It may seem paradoxical to say that what characterizes a person is external to him/her but, as Debray (2001) states, “le don de la prothèse fait l’humain de l’homme, lequel s’humanise en extériorisant ses facultés dans un processus d’objectivation sans fin” (the prothesis makes the ‘human’ from the “man”, humanizing and exteriorizing his faculties in an unending process of objectivization).
- The second shared idea is that even elementary tools deeply condition human activity: “the development of mathematics has always been dependent upon the material and symbolic tools available for mathematics computations” (Artigue, 2002). Even some very basic tools can have important effects: Lavoie (1994), for instance, shows the consequences of the introduction of the iron quill (instead of the goose quill) for the learning of arithmetic in the 19th century (writing more easily allowed pupils to do longer computations by hand and this lead to an earlier introduction of arithmetic in curricula). Tools *shape* the environment: “tools wrap up some of the mathematical ontology of the environment and form part of the web of ideas and actions embedded in it” (Noss and Hoyles, 1996, p. 227).
- The third shared idea is that the use of even elementary tools create “automatisms” and *routine procedures* (such as Bourdieu, 2003, describes for the case of traditional cereal management in North Africa). How to control these automatisms is a real question (see Section 4), especially in learning processes.

1.2. Some Oppositions

1.2.1 Human and Machine vs. Mind and Machine (or: what about the hand?)

One can distinguish first an opposition between Western and Eastern cultures, about the work of the *hand* and the work of the *mind:*

– Western culture establishes a structural opposition between human activities considered “manual” and those considered “intellectual", where primacy is given to the intellectual, that is, that which happens in the head, over work performed by any other part of the body (Bosch and Chevallard, 1999).

– In the opposite corner, Eastern culture considers there to be a dialectic interaction between hand and mind, as in the following quote, which describes the gradual synthesis of "proper gestures", a process which the accomplished adult no longer recognizes for all its complexity:

  Entre force et douceur, la main trouve, l'esprit répond. Par approximations successives, la main trouve le geste juste. L'esprit enregistre les résultats et en tire peu à peu le schème du geste efficace, qui est d'une grande complexité physique et mathématique, mais simple pour celui qui le possède. Le geste est une *synthèse* (...). L'adulte ne se rend plus compte qu'il lui a fallu accomplir un travail de synthèse pour mettre au point chacun des gestes qui forment le soubassement de son activité consciente, y compris de son activité intellectuelle. Il ne voit plus ce fondement et ne peut par conséquent plus le modifier (Tchouang Tseu, in Billeter, 2002).

Obviously, this is not only an opposition between two "geographical" cultures (Western and Eastern), but also an opposition between two philosophical points of view, an idealistic one and a dialectic one, as pointed out by Noss and Hoyles (1996, p. 52). Vygotski (1934), as an illustration of the dialectic point of view, evoked this sentence of Francis Bacon (1600): "Nec manus, nisi intellectus, sibi permissus, multam valent: instrumentis et auxilibus res perficitur" (human hand and intelligence, alone, are powerless: what gives them power are *tools* and assistants provided by culture). I will adopt this point of view, and from this moment speak of interactions "human/machine", implying human actions including both gestures and thought. For instance, there are not three separate types of computation (by hand, by mind and with a calculator): computing with a calculator mobilizes the calculator obviously, but also one or two hands and a mind.

1.2.2 Mind brain-independent vs. brain-dependent (or: is mind a pure spirit?)

A pure cognitivist point of view (Houdé et al., 1998, p. 84) considers the relationships between brain and mind as the relationships between hardware and software in a computer. According to this point of view, the science of the mind would be a special science, the science of thought. On the contrary, one can consider that the understanding of the mind's working needs to take into account the different levels of functional organization inside the nervous system. I will adopt this second point of view, which is that of Changeux (2002) or Houdé et al. (2002). Changeux develops the hypothesis of the existence, inside the brain, of two major neural networks: one, a treatment network, composed of parallel processors and the other a network dedicated to the supervision functions, including decision-making, goal-oriented behavior and systematic planning. Houdé observes that, during a logical deductive task, our brain can spontaneously work *economically*, i.e., mobilizing only the first network, perceiving words, forms and space.

This allows us to understand the following phenomenon (Guin and Trouche, 1999) where students' answers to the question "Does the f function defined by $f(x) = \ln x + \sin x$ have a limit $+\infty$ as x approaches $+\infty$?" were strongly dependent on the environment.

Whereas elementary theorems allow answering *yes* to the question, if students have a graphic calculator, due to the oscillation of the observed graphic representation (Figure 1), 25% of them answered no. Within a group of students of the same level without a graphic calculator, only 5% of wrong answers were collected. The students' work is thus altered by the multiplicity of easily available commands, furnishing a variety of views of this function.

The mobilization of the supervision network that Changeux speaks of requires clarification. I will call the individual mobilization of this network the *command process*.[3] This process is very important in a calculator environment, which allows a lot of gestures, makes available a lot of images and thus favors a lot of routine procedures.

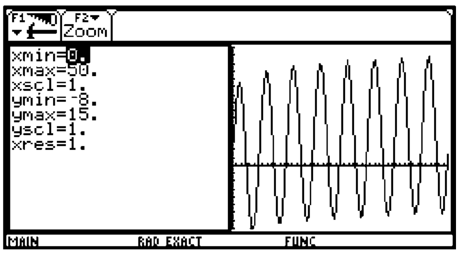

Figure 1. A representation of the function $x \mapsto \ln x + 10 \sin x$.

2. A NEW THEORETICAL FRAMEWORK: THE INSTRUMENTAL APPROACH

We will now study human/machine interactions in relation to computerized learning environments (CLE). The notion of CLE requires some explanation:

- The *environment* metaphor is important, putting in evidence the questions of objects' *viability*;
- Speaking of a *learning* environment is not neutral, it insists on *learners'* initiative and activity;
- We will use the expression *computerized* in an extended sense of an environment with software resources available for sustaining learners activity. In this sense, a classroom with calculators can be considered as a CLE.

Lagrange et al. (2003) discussed the relevance (with regard to CLE research) of a multidimensional research approach, using different theoretical frameworks. Here, however, I will choose only one approach, the *instrumental approach*, and extract from it all the theoretical tools relevant for our study here, that is, human–machine interaction. Obviously, this approach has not been chosen at random, it has been already used in recent research with some interesting results, and has been introduced in several papers, in particular in the last CAME Symposium by Michele Artigue (Artigue, 2002; Ruthven, 2002). I begin by clarifying some points.

2.1. Distinguishing between Artifact and Instrument

Recent work in the subject of cognitive ergonomy gives us theoretical means to understand human/machine interaction better. Verillon and Rabardel (1995) stress the difference between an *artifact* – a *given* object – and an *instrument* as a psychological *construct*: "the instrument does not exist in itself, it becomes an instrument when the subject has been able to appropriate it for himself and has integrated it with his activity". More precisely, an instrument can be considered as an extension of the body, a functional organ made up of an artifact component (an artifact, or the part of an artifact mobilized in the activity) and a psychological component. The construction of this organ, named *instrumental genesis*, is a complex process, *needing time,* and linked to the artifact characteristics (its *potentialities* and its *constraints*) and to the subject's activity, his/her knowledge and former method of working.

The psychological component is defined through the notion of a *scheme*. Vergnaud (1996) has redefined a scheme, first introduced by Piaget (1936), as the "invariant organization of behavior for a given class of situations", a dynamic functional entity. In order to understand a scheme's function and dynamics, it is necessary to consider all of its components: the goals and the anticipations, the rules of action, gathering of information, control-taking and the *operative invariants*. The operative invariants are the *implicit* knowledge contained in the schemes: *theorems-in-action*, that is, propositions believed to be true. A scheme has thus three main functions: a *pragmatic* function (it allows the agent to do something), a *heuristic* function (it allows the agent to anticipate and plan actions) and an *epistemic* function (it allows the agent to understand what he is doing).

It is important to distinguish between gestures and schemes. Perhaps a metaphor may help to clarify the difference: a scheme could be compared to an iceberg, the emerged part being the gestures (elementary behavior that may be observed), the submerged part being constituted of operative invariants. A scheme is the psychological locus of the dialectic relationship between gestures and operative invariants, that is, between activity and thought. Operative invariants involved in the scheme guide the gestures and, at the same time, the repetition of such gestures, in a given environment, installs in the mind a particular knowledge.

Let us consider the following example (Drijvers, in Guin et al., 2004), a trace of the scheme "Isolate–Substitute–Solve" (ISS, see Figure 2). This scheme allows the students to solve a system of two equations with two unknown. It can appear as a sequence of gestures

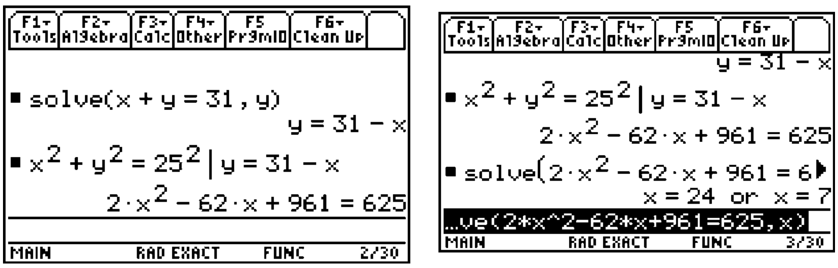

Figure 2. A trace of the ISS scheme on the TI-89.

on the keypad of the calculator, but it requires considerable knowledge, although not necessarily the same knowledge for each student.

For example, ‘‘the fact that the same *solve* command is used on the TI-89 for numerical solutions and for the isolation of a variable requires an extended conception of *solve*: it also stands for *taking apart a variable* and for *expressing one of the variables in terms of one or more others in order to process it further*’’ (Drijvers in Guin et al., 2004, p. 227). The ISS scheme requires the second conception. Thus the ISS scheme is constituted both by observable gestures and by knowledge involved in the gesture making.

2.2. Distinguishing Different Types of Schemes

Rabardel (1995) introduced the notion of *utilization scheme* of an artifact, which he describes as a scheme organizing the activity with an artifact associated with the realization of a given task. He distinguishes between two kinds of utilization schemes, *usage schemes* oriented towards the management of the artifact (turning on a calculator, adjusting the screen contrast, choosing a particular key, etc.) *and instrumented action schemes*, entities oriented to the carrying out of specific tasks (computing a function’s limit, for example).

It may seem surprising to call something that could appear as a simple gesture a usage *scheme* (i.e., something associated with operative invariants). Nevertheless, even a simple gesture produces *and* results from some knowledge. We have, for instance, found evidence (Guin and Trouche, 1999) of the importance of a particular usage scheme, *the approximate detour*, which consists of a combination of keystrokes, which returns, when working with a symbolic calculator in ‘‘exact’’ mode, an approximate value of a symbolic expression. This usage scheme can be seen as a simple gesture, oriented towards the management of the calculator. Looking beyond the simple gesture on the calculator keypad involves considering the gesture not as an isolated case but as a component of the instrumented action schemes implemented by the student in order to solve the given task. The various functions of the approximate detour scheme include:

- *Anticipation or checking* function (obtaining an approximated value may be a step in a process towards obtaining a proof);
- *Determination* function (the obtained approximated value is considered as a final result).

Observation of students' work indicates regularities in the functions for which this scheme is employed (Trouche, in Guin et al., 2004): for some students, the approximate detour always has a determination function, for other ones it always has an anticipation or checking function. In other words it contributes to building different kinds of knowledge about, say, the real numbers. Underlying this scheme, there are different operative invariants: it is useful to consider such actions in relation to schemes, and not as simple gestures. Observing such regularities requires that usage schemes are considered as components of larger entities, instrumented action schemes.

2.3. Social Schemes or Individual Schemes?

Rabardel and Samurçay (2001) define *social schemes* in the following way: "they are elaborated and shared in communities of practice and can give rise to appropriation by the subjects, even come under training processes". This definition allows us to move beyond a former opposition between two theoretical approaches: genetic epistemology (Piaget, 1936), with its focus on the world of nature, and mediation theories (Vygotski, 1934), focused on the world of culture.[4] A scheme, according to Piaget, is, for a subject, a *means of personal assimilation* of a situation and objects s/he is confronted and, at the same time, Rabardel and Samurçay (2001) insist on this point, a scheme is itself *the product of an assimilation* activity, in which the environment – and the artifacts available – play a major role. Artifacts always carry a social element: they are products of social experience: "Tools are not passive, they are active elements of the culture into which they are inserted" (Noss and Hoyles, 1996, p. 58). From this point of view it is impossible to distinguish, on the one hand cognitive structures (schemes) and on the other hand, cultural systems: schemes always have a social part and instrumental genesis always has individual and social aspects. In this sense, the notion of *social scheme* is very close to the notion of *situated abstraction*, defined by Noss and Hoyles (1996, p. 121) "as a complex construction – being a product of activity, context, history and culture".

The "balance" between the individual and social aspects depends on:

- Material factors (it is obvious that the intimacy of calculator screens as compared to computer screens favors individual work whereas computer screens are more conductive to small group work);

- Artifact availability (students only use calculators in the mathematics class, or sometimes they are loaned to them for the whole year, or sometimes they belong to them);
- The attitude to and use of the artifact by the teacher and the integration s/he builds into classroom activities (see Section 4 below).

Therefore an instrument is the result of a construction by a subject, in a community of practice, on a basis of a given artifact, through a process, the instrumental genesis. An instrument is a mixed entity, with a given component (an artifact, or the part of an artifact mobilized to realize a type of task) and a psychological component (the schemes organizing the activity of the subject). All schemes have individual and social aspects. There are two levels of schemes: usage schemes, directed towards the managing the artifact, and instrumented action schemes, oriented by the activity itself.

3. TWO PROCESSES CLOSELY INTERRELATED

I consider now instrumental genesis itself in more depth. This can be seen as the combination of two processes (see Figure 3):

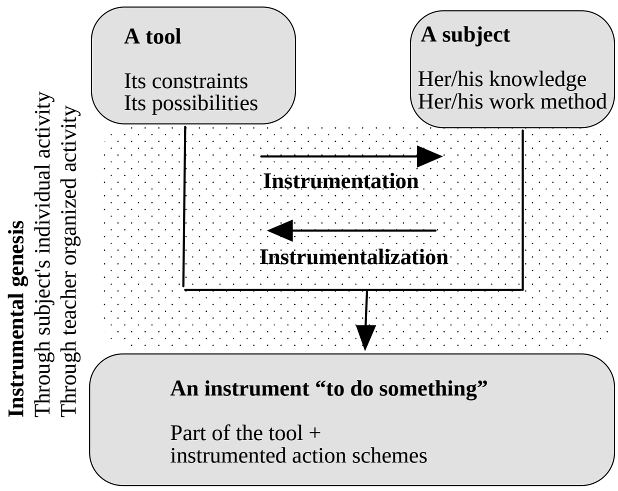

Figure 3. Instrumental genesis as a combination of two processes.

– An *instrumentalization process* (directed towards the artifact);
– An *instrumentation process* (directed towards the subject).

3.1. *The instrumentation process*

As Noss and Hoyles (1996, p. 58) note: "Far from investing the world with his vision, the computer user is mastered by his tools". It is important to give a name to this mastering process[5]: *instrumentation* is precisely this process by which the artifact *prints its mark* on the subject, i.e., allows him/her to develop an activity within some boundaries (the constraints of the artifact). One might say, for example, that the scalpel *instruments* a surgeon. This is obviously the case in every CLE.

To understand this process we first need to study the *constraints* and "*enablements*"[6] of an artifact. In the case of a CLE, these constraints are linked to *computational transposition*, described by Balacheff (1994) as "the work on knowledge which allows a symbolic representation of it and the implementation of this representation by a computer system". In order to analyze this transposition, we distinguish between three types of constraints (Guin and Trouche, 2002):

- *Internal constraints* intrinsically linked to the hardware;
- *Command constraints* linked to the existence and to the form (i.e., the syntax) of the various commands;
- *Organization constraints,* linked to the organization of the keyboard and, more generally, of the interface between the artifact and the user.

Box 1 provides an illustration of this typology for a particular calculator. It is, of course, possible to question the nature of a given constraint with regard to one of these three defined types. The purpose of this typology of constraints, however, is not to strictly define a totally self-contained set of categories but, rather, to make it easier for the teacher or researcher to undertake an a priori analysis of different ways of performing particular tasks made possible by the artifact.

In a graphic calculator environment, artifact constraints can thus contribute to the building of specific schemes: obviously the organization constraints of graphic calculators favor the graphical study of functions (most of the time, the keys for numerical study of functions are located "behind" the keys for graphical study). Trouche (2003)

Box 1

Some constraints of a symbolic calculator for limit computation.

(Calculator used: Texas Instruments TI-92)

1) Internal constraints (what, by nature, can the artifact do?)

A symbolic calculator contains a CAS application (Computer Algebra System); it can determine an exact limit provided that the corresponding "knowledge" has been entered. In Figure (a) there is a mathematically well-defined limit, but the result is not "known" by the calculator's algorithms: it answers "undef" to the question.

A symbolic calculator can also (like a graphic calculator) give graphic or numerical information on the local behavior of a function. The processing is done by numerical computation.

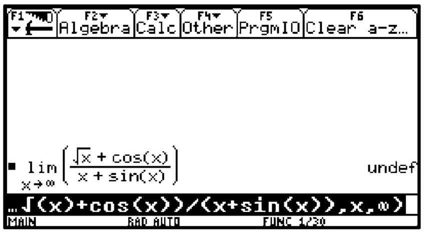

Figure a

2) Command constraints (what are the available commands?)

There is only one command for a limit computation, it is located in the formal calculus application, see Figure (b). The syntax of this command is "limit(f(x),x,a)" which corresponds to the order in the statement "the limit of f(x) when x tends to a".

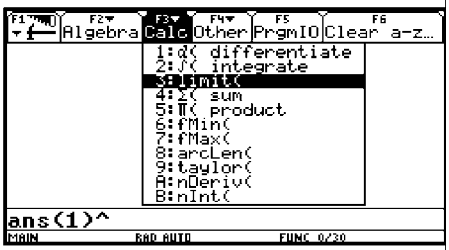

Figure b

Nevertheless this command may be combined with the *approximate detour (*see 2.2). One can verify (Figure c) that this "limit" command, applied to the function f(x) = (cos x)` does not give a result directly but instead the result is obtained by switching to approximate computation (3rd line on the screen).

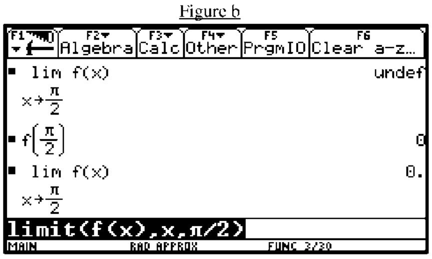

Figure c

3) Organization constraints (how are the available commands organized?)

The different applications (symbolic, graphical or numerical) allowing the study of functions are directly accessible from the keyboard. With the graphical or numerical applications, the calculator user must first choose the interval of x and then the interval of y. This is a natural order for the study of functions, but it is not an adequate order to study limits: in this case mathematical organization and artifact organization are opposed from a chronological point of view.

describes the behavior of one particular student as follows: he grabs his calculator and ''enters'' the function to be studied in the function editor then infers the answers from the shape of the function graphic (cf. Section 1.2) choosing window with large values for x. By observing the student resolving a set of tasks of a similar type one may ask him to state his choices explicitly, thus the teacher or researcher can attempt to determine the operative invariants linked to these gestures and orienting the student's answer.

The student in this example may explain, for instance, that considering the function graphic allows him to conclude the following: ''if the function increases with great speed it is okay. On the other hand, if the function starts to decrease or oscillate then it is not good.'' Consequently, one can put forward the hypothesis that the student's scheme integrates *theorems-in-action* (cf. Section 2.1) of the following type ''if the function increases very strongly, then the limit of f is infinite'', ''if the limit of f is infinite, then f is necessarily increasing''. From all these properties emerges a conception of the type: ''to say that f has an infinite limit means that, when x is large, $f(x)$ is very large, and keeps getting bigger without oscillating too much''; this may be classified as a situated abstraction (cf. Section 2.3).

Obviously, even if a particular environment *favors* a particular scheme, this does not mean that there is *automaticity* within a given technological environment towards a given scheme, as the following discussion will show.

3.2. Differentiation of Instrumentation Processes, Depending on the Environment Complexity

Trouche (in Guin et al., 2004) shows the possible differentiation of students' behaviors within the same environment. We have also noticed that, the more complex the environment, the larger the differentiation of instrumentation processes. The behaviors of two students searching for a function limit unknown by the calculator (see Box 1, Figure (a)), in a symbolic calculator environment (which is more complex than a graphic calculator environment, see Box 1), provide an example of this differentiation. We have summarized below (see Box 2) the two sets of results. Clearly, the two processes of instrumentation are not the same as in a graphic calculator environment. For both of the students, using the command ''limit'' is the first gesture. Thus the conception of ''limit'' is not necessarily associated with ideas of functions increasing or decreasing, nor with conditions of regularities. A limit could even have only the sense of "being produced by a particular key of the calculator".

What appears mainly is the great difference between the two instrumentation processes for the two students (see Box 2):

- Student 1 can articulate all the artifacts available in the environment (calculator, paper–pencil, and theoretical results);
- Student 2 uses only one key (the limit key) of only one application of only one artifact. (The more complex is the artifact, the more simple seems the activity.)

In the first case, we observe a strong command process, which enables the Student to control an efficient instrument. In the second case, the command process appears much weaker: Student 2 is not able to move forward in his understanding of limits, with regard to the given task, in this kind of CLE.

3.3. The Instrumentalization Process

This process is the component of instrumental genesis directed towards the artifact. Instrumentalization can go through different stages: a stage of discovery and selection of the relevant functions, a stage of personalization (one fits the artifact to one's hand) and a stage of transformation of the artifact, sometimes in directions unplanned by the designer: modification of the task bar, creation of keyboard shortcuts, storage of game programs, automatic execution of some tasks (calculator manufacturers' websites and personal websites of particularly active users often offer programs for certain functions, methods and ways of solving particular classes of equations, etc.). Instrumentalization is a differentiation process directed towards the artifacts themselves.

Rabardel (1995) chooses the word "catachresis" to designate a situation where an artifact is used in place of another one, or to do something it was not conceived for. For example, it is well known that some students use their calculators simply to store a lot of mathematical results (computations, rules, theorems, ...). At least two reactions to this practice are possible for the teacher:

- Firstly, s/he can say: a calculator is not made for this kind of use. So students are not allowed to do that, or computers will not be allowed during the examinations;

Box 2

Studying $\lim_{x\to+\infty} \frac{\sqrt{x}+\cos x}{x+\sin x}$: using the same artifacts (TI-92), two very different approaches

Student 1

He defines first the function *f* for the calculator (see Figure 4) "*then I could avoid writing several times this complex thing*". Calculator answer: **undef.**

"*Oh, these functions sine and cosine often cause trouble when looking for limits, I need to get rid of them*".

On his paper, he frames sines and cosines between – 1 and +1, then frames the function *f*, for x > 0:

$$\frac{\sqrt{x}-1}{x+1} \le f(x) \le \frac{\sqrt{x}+1}{x-1}$$

He uses his calculator to find the limits of the left and right function: 0.

"*According to the theorem about functions limits and inequalities, I can say that my function f also has 0 as a limit*".

"*Let us have a glance at the graphs of the three functions.* He plots the three functions: "*the function f is well constrained by the two others in the neighborhood of +∞*".

Then: "*I can also change the variable*". On paper:

$X=\sqrt{x},\quad f(X)=\frac{X+1}{X^2+1}$

"*I can use the theorem about polynomial functions, or do some factorisations and use the theorems about limits and operations*":

On paper again: $\frac{X+1}{X^2+1}=\frac{1+\frac{1}{X}}{X+\frac{1}{X}}$

Summary of the work (1 hour): paper-pencil and calculator articulated, a multi-register work (algebraic and graphical studies), expression and construction of knowledge about limits, a rich limit scheme.

The artifact complexity is mastered and contributes to enrich the instrumentation process and to build an efficient instrument for function limit study.

Student 2

He uses the command limit of the CAS application, applied to the given function.

Calculator answer: **undef**

« *Oh, I made a mistake in writing the command!* »

He writes again, same calculator answer.

« *Oh, I am so weak, I have to try again* » (writing the function $\frac{\sqrt{x}+\cos x}{x+\sin x}$ takes a long time). Same calculator answer…

« *Oh, I have understood, the calculator doesn't know the f function, I have to define it!* ».

He defines the function f (see figure 4 below).

Again the command limit, again the answer **undef**.

New perplexity, and new idea: "*when a limit isn't defined, it is sometimes possible to look at the left, or at the right of the point. So I am going to look at the right of +∞, so I will be as far as possible*" (see Figure 4). Always answers **undef**.

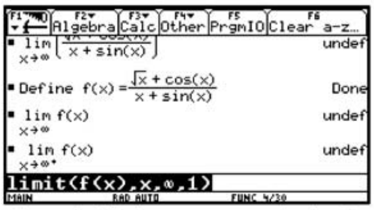

Figure 4. *Calculator screen copy of student 2.*

At last, he breaks down the problem into sub-problems, looking for the limits of $\sqrt{x}$ ("*It works, I obtain +• as a limit!*") and of sin x and cos x ("*there is the problem: these two functions have no limit, it is the reason why my function f has no limit*").

Summary of the work (1 hour 30 mins): no paper used, a single register work (no numerical nor graphical studies), no idea of the function behavior, a quite weak limit study scheme.

The artifact complexity doesn't contribute to help the student's activity and to build a efficient mathematical instrument.

> Secondly, s/he can say: it is a particular use of this artifact. How can it be organized, structured, integrated into the student's mathematical practice?

Underlying these two reactions, there are two ideas of what an instrument is. According to the instrumental approach, a catachresis can be considered as the expression of a subject's specific activity: what a user thinks the artifact was designed for and how it should be used. This highlights a very important idea: *the user's conception of the instrument is formed through use*. This idea is all the more important for CAS, which was not initially conceived for learning (or teaching); thus the process of conception of CAS by teachers and students can be seen as a loop: analyzing the constraints, integrating into an environment, analyzing the uses, defining new specifications, etc. (Lagrange and Py, 2002). This approach corresponds to a new paradigm and a movement from a "techno-centered" point of view (which prevails as the starting point for software such as Intelligent Tutoring Systems) to an "anthropocentric" one.

So two processes, closely connected, make up the instrumental genesis. In fact, it is not possible to clearly distinguish between these two processes, to say, for example "that is an instrumentation scheme", or "that is an instrumentalization scheme". All activity is oriented by some goals linked to the realization of particular tasks, so we can talk about an *instrumented action scheme*, with the understanding that this instrumented action scheme bears the marks of the two processes. The interest of this distinction is in the dialectic between the two protagonists of the instrumental genesis, the subject and the artifact.

So at the outcome of its genesis, an instrument is constituted, with regard to its material components, by a part of the artifact – modified from its initial state – and with regard to its psychological components by schemes built by the subject, relative to the execution of specific tasks. In fact, the situation is a little more complex. A student has, at his/her disposal, a set of artifacts (paper/pencil, rule, compass, calculator). A symbolic calculator is itself a set of various artifacts (CAS, spreadsheet, word processing, etc.). This set will provide for each student the subject materials for several instruments, related to several types of tasks. The *articulation* of these instruments demands a command of the process which is not easy to build (Artigue, 2002, speaks of "the unexpected complexity of instrumental genesis"), and requiring assistance from the teacher, which we are now going to examine.

4. INSTRUMENTAL ORCHESTRATION, AS A GUIDE FOR INSTRUMENTAL GENESIS

4.1. A Definition

The word *orchestration* is often used in the cognition literature. Dehaene (1997) uses this word in relation to an *internal* coordination function of distributed neural networks. Ruthven (2002) also uses the word, in the mathematical education field, to refer also to an internal cognitive function (in the context of the teaching and learning of the derivative concept): "unifying ideas are careful orchestrations of successive layers of more fundamental ideas around a more abstracted term". In fact, the necessity of orchestrations, in the sense of an internal coordination function, clearly manifests itself in the mathematical sciences, where learning can be seen as "the construction of a *web* of connections – between classes of problems, mathematical objects and relationships, real entities and personal situation-specific experiences" (Noss and Hoyles, 1996, p. 105). The word *orchestration* is indeed quite natural when speaking of a set of *instruments*.

I introduce the term *instrumental orchestration* to point out the necessity (for a given institution – a teacher in her/his class, for example) of *external steering* of students' instrumental genesis. This necessity is rarely taken into account. One can find in textbooks or papers relating to CLE experiments comments on material components (calculators or computers, kinds of software, overhead projectors, directions for use) and on didactical components (exposition of the mathematical subject and of the different stages of treatment), but seldom information about the environmental organization, that is on the organization of the students' or teachers' work space and time.

An instrumental orchestration is defined by *didactic configurations* (i.e., the layout of the artifacts available in the environment, with one layout for each stage of the mathematical treatment) and by *exploitation modes* of these configurations. For each orchestration, the *main objectives*, originating from the necessity of orchestration itself and the *secondary objectives,* linked to the chosen exploitation modes, should be distinguished. The configurations and their exploitation modes produce *accounts* of activity (i.e., to say results of the activity which can be observed by persons other than the subject involved in this activity). The socialization of these accounts (research reports, calculators screens, etc.) is essential: ''The production, interpretation and negotiation of accounts plays an extremely important part in the development of personal schemes (cognitive structures) and their co-ordination with diamathematical techniques (cultural systems)'' (Ruthven, 2002), that is, according to my definitions, the development and coordination of the social part of schemes.

Instrumental orchestrations can act at several levels:

– The first level (that of the artifact itself);
– The second level (a psychological one) of an *instrument* or a *set of instruments*;
– The third level (a ''meta'' one) of the *relationship* of a subject with an instrument or a set of instruments.

These three levels correspond to different types of artifacts, which Wartofsky distinguishes as follows:

– ''Primary artifacts, corresponding to the concept of the artifact as it is ordinarily utilized;
– Second level artifacts, which correspond both to representations and action modes utilizing first level artifacts;
– Third level artifacts, notably for trained persons, corresponding to the social and cognitive development by simulation situations and reflective methods of activity self-analysis, both individual and collective'' (Wartofsky, 1983).

Example of first and of third level orchestration can be found in (Trouche, in Guin et al., 2004):

– The first level example concerns software issues (on a symbolic calculator); it is a matter of helping students in computing and, more to the point *understanding* the limits of functions. It constitutes a particular answer to the question asked by Hoyles: ''we need software where children have some freedom to express their own ideas, but *constrained* in ways so as to focus their attention on the mathematics (Hoyles, 2001);
– The third level example is a self-analysis device, which aims to provide students the ability to reflect on their own-instrumented activity by providing them with observable traces of it.

In the following, we present an example of a second level orchestration.

4.2. An Example of Second Level Instrumental Orchestration

The school utilization of individual artifacts, calculators fitted with a small screen, poses the problem of the socialization of students' actions and productions. For such a socialization to occur, particular arrangements are required. Since the beginning of the 1990s, there has been for each type of calculator, an artifact – a view-screen – which allows the projection of the calculator's screen onto a large screen for the entire class to see. Guin and Trouche (2002) present an instrumental orchestration which exploits this arrangement and whose main objective is the socialization – to a certain extent – of students' instrumental geneses.

The configuration of this orchestration (see Figure 4) rests on the devolution of a particular role to one student: this student, called the sherpa student,[7] pilots the overhead-projected calculator. S/he will thus be used, by both class and teacher, as a reference, a guide, an auxiliary and a mediator. This orchestration favors collective management of a part of the instrumentation and instrumentalization processes: what a student does with her/his calculator – the traces of her/his activity – are seen by all, allowing the comparison of different instrumented techniques and giving the teacher information about the instrumented actions schemes being built by the sherpa-student.

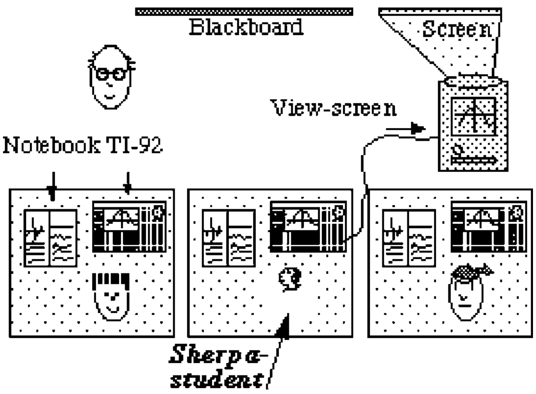

Figure 4. The sherpa-student, part of an instrumental orchestration.

It also presents other advantages:

– The teacher is responsible for guiding, through the student's calculator, the calculators of the whole class (the teacher does not perform the instrumented gesture but checks how it is performed by the sherpa-student). The teacher thus fulfils the functions of an orchestra conductor rather than a one-man band;[8]
– For his/her teaching, the teacher can combine paper–pencil results obtained on the board, and results obtained by the sherpa-student's calculator on the class screen. This facilitates, the combination of "paper/pencil" and calculator work by the students themselves on their own desks.

Several exploitation modes of this structure can be considered. The teacher can first organize work phases of different kinds:

- Sometimes calculators are turned off (and so is the overhead projector): it is then a matter of paper/pencil environment work;
- Sometimes both calculators and overhead projector are turned on and work is strictly guided by the sherpa-student under the supervision of the teacher (students are supposed to have exactly the same thing on their calculator screens as is on the projector screen). Instrumentation and instrumentalization processes are then strongly constrained;
- Sometimes calculators are on as well as the overhead projector and work is free for a given time. Instrumentation and instrumentalization processes are then relatively constrained (by the type of activities and by referring to the sherpa-student's calculator which remains visible on the big screen);
- Sometimes calculators are on and the projector is off. Instrumentation and instrumentalization processes are then only weakly constrained.

These various modes seems to illustrate what Healy (2002) termed *filling out* and *filling in*, in the course of classroom social interaction:

– when the sherpa-student's initiative is free, it is possible for mathematically significant issues *to arise out* of the student's own constructive efforts (this is a filling out approach);
– when the sherpa-student is guided by the teacher, it is possible for mathematically significant issues to become *appropriated during* the student's own constructive efforts (filling in approach).

Other variables in the situation must also be considered: will the same student play the role of the sherpa-student during the whole time or, depending on announced results, should the calculators of different students be connected to the projector table? Should the sherpa-student sit in the first row or should she/he stay at her/his usual place? Do all students play this role in turn or should only some of them be privileged?

Depending on the didactic choices made, secondary objectives of this orchestration can arise:

- To favor debates within the class and the making explicit of procedures: the existence of another reference different from the teacher's allows the development of new relationships between the students and the teacher, and between the sherpa-student and the teacher – about a result, a conjecture, a gesture or a technique;
- To give the teacher a means to reintegrate remedial or weak students into the class. The sherpa-student function actually gives remedial students a different status and forces the teacher to tune his/her teaching procedures onto the work of the student who is supposed to follow her/his guidelines, follow-up work of this student on the projector screen allows very fast feedback actions from both teacher and class.

This instrumental orchestration partakes in the coordination of all the classes' instruments and it favors the articulation, for each individual, of different instruments within her/his mathematical work.

4.3. About Metaphors for Computational Settings

Other metaphors have been used about computational settings, such as *scaffolding*, or *webbing* (Noss and Hoyles, 1996):

- Scaffolding originates from Vigotskyan theory, it is "the graduated assistance provided by an adult which offers just the right level of support so that a child can voyage successfully into his/her zone of proximal development" (Noss and Hoyles, 1996, p. 107). Hoyles and Noss (1987) extend this metaphor to computational settings, the computer playing the same normally ascribed to a human tutor;
- "The idea of webbing is meant to convey the presence of a structure that learners can draw upon and reconstruct for support

– in ways that they choose as appropriate for their struggle to construct meaning for some mathematics" (Noss and Hoyles, 1996, p. 108).

These various metaphors do not operate at the same level: scaffolding and webbing are essentially related to internal processes, orchestrations are mainly related to external organizations. But some comparison could be useful:

- In scaffolding and in orchestration, there is an idea of structure erected around the learner by an external agency; in scaffolding, the computer could be this structure, in orchestration, the teacher's role is pointed out. In webbing no place for such an external agency is emphasized;
- Scaffolding is mainly related to the support of skills in practice, whereas webbing and orchestration are mainly applied to the learning of *conceptual fields* (Vergnaud, 1990);
- Scaffolding seems to be domain-independent, whereas webbing and orchestration are domain-dependent;
- Scaffolding is associated to a conception of knowledge as structured and hierarchical, whereas webbing and orchestration take into account the fluidity and flexibility of computational settings;
- In scaffolding, some mathematical progress may be independent of the student's own construction, whereas in webbing and orchestration understanding emerges from connections which are forged in use by the user.

Finally, the philosophies underlying both webbing and orchestration appear quite similar. There nevertheless remains an important difference: instrumental orchestration points out the teacher's role, whereas in webbing this role does not clearly appear.

4.4. Some Consequences

This understanding of the nature of orchestrations, adapted to mathematical work at various stages, is quite recent. If one rereads some "old" papers, it seems that insufficient attention was given to such orchestrations. For example, in Trouche (1998), I presented an activity for a 12th grade class, in a symbolic calculator environment (see Box 3). Its didactic goal was the shaping of the concept of infinite limit, through work on comparing the limits of power functions and exponential functions.

Box 3

Exposition of a problem (Trouche, 1998)

Consider the equation $e^x = x^{10n}$, n being a positive integer.

Convention: We will call the "first equation" the equation obtained with n = 1, the "second equation" the equation obtained with n = 2, etc.

Notation: Call a_1, b_1, c_1... the solutions, in order, of the first equation, a_2, b_2, c_2... the solutions, in order, of the second equation, etc.

Answer the following questions in any order.

* Solve the first, second, third and tenth equations; give an approximate value (with a precision of 10^{-5}) for each solution.

* Solve the n^{th} equation $e^x = x^{10n}$.

* The solutions a_1, a_2, ..., a_n... constitute a sequence. The same for the second sequence of solutions (b_n) of successive equations, etc. Observe the sequences (a_n), (b_n), (c_n)... What conjectures can you make about their growth?

This activity is quite complex and it requires various instruments: an equation solving instrument, a function variation study instrument, a limit computation instrument, a sequence variation study instrument (see Figure 5). For all the students, the genesis process of these instruments is still ongoing. What is more, making use of each instrument needs a strong command process: the TI-92 command "Solve" gives only two solutions (instead of the three solutions which

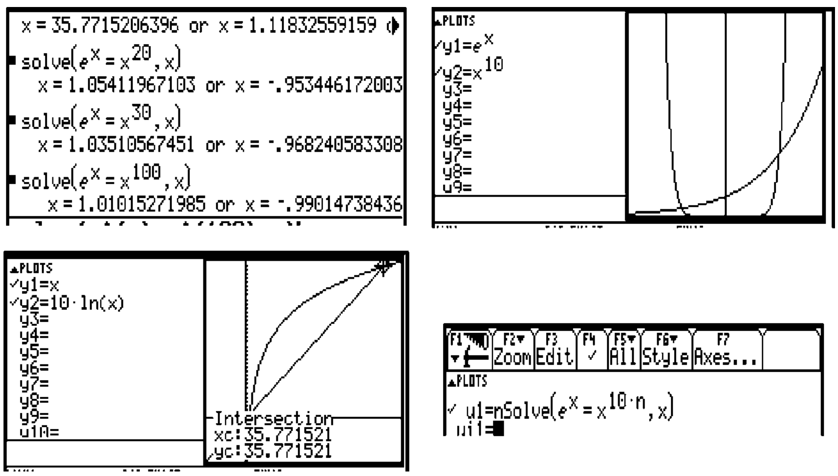

Figure 5. Some screen displays obtained during this activity.

exist) and the command ‘‘graph’’ suggests that there are, indeed, only two solutions. Finally it is not only a matter of using these instruments separately but on the contrary, to articulate them, that is to build coherent instrument systems from a set of artifacts.

..In this orchestration, the configuration is a ‘‘practical’’ (students work in pairs, without the teacher’s help) and students have to give a written research report at the end of the session. It works because of the very particular conditions of this experiment: an expert teacher, part of a research team, clever students, all equipped with the same calculator and strongly motivated by participating in a pilot experience. But it has been impossible to transpose this kind of activity in ‘‘normal classes’’. In my opinion, one of the main reasons for this situation is the weakness of reflection about the instrumental orchestrations which necessarily accompany this activity.

This activity is indeed very dense. Its organization should have been preceded by a study of its structure and a decomposition in different stages: a first stage which allows students to understand the problem and appropriate it as their own, a second stage for the exploration of some particular examples, a third stage to discuss various conjectures. Each of these stages needs a particular orchestration, that is, a configuration and its specific exploitation modes (e.g., a sherpa-student configuration during the appropriation of the problem, a practical configuration for the explorations, a ‘‘colloquium’’ configuration to discuss conjectures, etc.). Such a sequence of stages may constitute a *scenario in use* of this activity (Allen et al., 1995), which may make appropriation by another teacher easier.

This leads to a new conception of pedagogical resources for CLE. In the context of distance training, Joab et al. (2003) present a new structure for these resources, including, for each activity, an identification sheet, a student sheet, a teacher sheet, a scenario in use, experimentation reports and a technical sheet. Underlying this new structure there is a conception of *a pedagogical resource as an artifact* for a community of practice, this artifact becomes an *instrument* through an instrumental genesis: the instrumentation process modifies the teachers’ behavior, the instrumentalization process modifies the resource itself (through the experimentation reports and the rewriting process).

Obviously, the conception of these resources cannot be done by a teacher alone, in his/her classroom. It requires a chain of ‘‘technical solidarities" (Chevallard, 1992), where computer engineers, didactical engineers and teachers play complementary roles.

CONCLUSIONS

I have presented a general framework for the instrumental approach, trying to show its relevance for CLE studies. The distinction between the artifact, which is given, and the instrument, which is built through activity, obliges us to pay attention to the instrumental genesis. This genesis is a complex process of construction of both usage schemes and instrumented action schemes. This process is in fact a combination of two closely connected processes, an instrumentation process, directed towards the subject and an instrumentalization process, directed towards the artifact. The schemes thus constructed always have a social dimension (because of the social aspect of each artifact and/or because of the context of the schemes' elaboration within a community of practice).

Very sophisticated artifacts such as the artifacts available in a CLE give birth to a set of instruments. The articulation of this set demands from the subject a strong command process. One of the key elements for a successful integration of these artifacts into a learning environment is the institutional and social assistance to this individual command process.

Instrumental orchestrations constitute an answer to this necessity. They assist individual command process in two ways:

- During the time of the orchestration itself;
- Beyond the orchestration (an instrumental orchestration gives birth to new instrument systems).

The constitution of the instrument systems is linked to the introduction of artifacts within mathematical practice. Rabardel (2000, p. 212) describes the necessity of didactic management of these instrument systems:

> "The introduction of a new artifact must, at the didactic level, be equally managed in its impact on previously built instrument systems. This issue appears particularly crucial to us in the present context of technological abundance. Which artifacts should we propose to learners and how can we guide them through instrumental genesis and along the evolution and balancing of their instrument systems? For which learning activities and which components of mathematical knowledge?".

Designing of instrumental orchestrations seems to give some elements of an answer to these questions. Finally, taking into account instrumental orchestrations opens new perspectives for conceptions of pedagogical resources for CLE. These perspectives are certainly very important at a moment given the abundance of web resources easily available but not necessarily easily integrated into one's practice.

ACKNOWLEDGMENT

Thanks to Dominique Guin, Lulu Healy, Phillip Kent and John Monaghan, who carefully read successive versions of this article and gave me precious advice.

NOTES

[1] This article originates from a lecture given at the Third CAME (Computer Algebra in Mathematics Education) Symposium (Reims, June 2003).

[2] Thinking in French and writing in English add difficulties; for instance, the French word "ordinateur" and the English word "computer" do not refer to the same thing: the French word refers to questions of *order*, while questions of *computation* are signalled by the English word.

[3] In Guin and Trouche (1999), we wrote: "The student's command process is characterized by the conscious attitude to consider, with sufficient objectivity, all the information immediately available not only from the calculator, but also from other sources and to seek mathematical consistency between them".

[4] In the same sense, Wallon (1949) said: "Each individual is genetically social".

[5] This point was discussed in the CAME Symposium 2001. Would it not be sufficient to note that "tools shape the environment"? In my opinion, this is a question of method: it is always important to *name* things we want to study (as in a process of equation solving, when we name the unknown *x*).

[6] We can distinguish for a given tool *constraints* (obliging the user in one way, >>?? empeaching<< in another way), *enablements* (effectively making the user able to do something), *potentialities* (virtually opening possibilities) and *affordances* (favoring particular gestures), which are closely interrelated.

[7] The term sherpa refers to the person who guides and who carries the load during expeditions in the Himalaya mountains, and also to diplomats who prepare international conferences.

[8] This advantage is not a minor one. We have shown that teachers, in complex technological environments, are strongly prone to perform by themselves all mathematical and technical tasks linked to the resolution of problems in the class; the view-screen is then used to project the teacher's screen.

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Department of Mathematics
LIRDEF, IREM & LIRMM
Université Montpellier II
Montpellier Cedex
France
E-mail: trouche@math.univ-montpz.fr