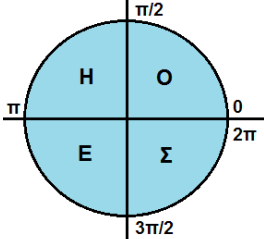
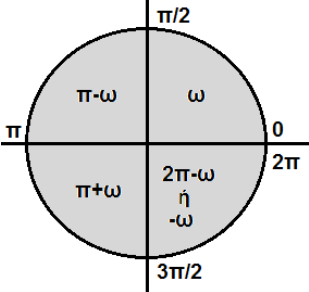
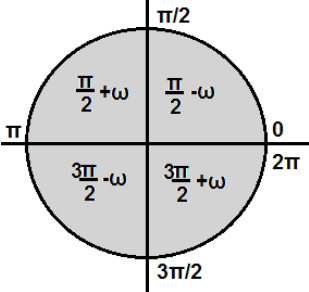


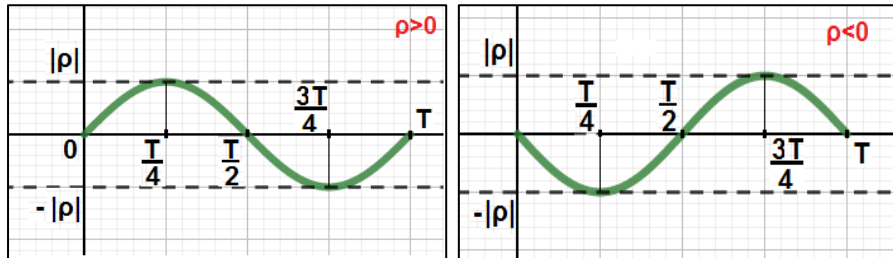
ΒΑΣΙΚΟ ΤΥΠΟΛΟΓΙΟ ΤΡΙΓΩΝΟΜΕΤΡΙΑΣ

Μοίρες – Ακτίνια	Πρόσημο τριγωνομετρικών αριθμών	
$\frac{\mu}{180^\circ} = \frac{\alpha}{\pi}$		<p>Ο : Όλοι οι τριγωνομετρικοί αριθμοί είναι θετικοί Η : Μόνο το ημίτονο είναι θετικό Ε : Μόνο η εφαπτομένη & συνεφαπτομένη είναι θετικές Σ : Μόνο το σνημίτονο είναι θετικό</p>
Βασικές Τριγωνομετρικές Ταυτότητες	Αναγωγή στο 1 ^ο Τεταρτημόριο	
$\eta\mu^2\chi + \sigma\upsilon\nu^2\chi = 1 \Leftrightarrow \begin{cases} \sigma\upsilon\nu^2\chi = 1 - \eta\mu^2\chi \\ \eta\mu^2\chi = 1 - \sigma\upsilon\nu^2\chi \end{cases}$ $\epsilon\varphi\chi = \frac{\eta\mu\chi}{\sigma\upsilon\nu\chi}$ $\sigma\varphi\chi = \frac{\sigma\upsilon\nu\chi}{\eta\mu\chi}$ $\epsilon\varphi\chi \cdot \sigma\varphi\chi = 1 \Leftrightarrow \begin{cases} \sigma\varphi\chi = \frac{1}{\epsilon\varphi\chi} \\ \epsilon\varphi\chi = \frac{1}{\sigma\varphi\chi} \end{cases}$ $\eta\mu^2\chi = \frac{\epsilon\varphi^2\chi}{1 + \epsilon\varphi^2\chi}$ $\sigma\upsilon\nu^2\chi = \frac{1}{1 + \epsilon\varphi^2\chi}$	 <div style="display: flex; justify-content: space-around;"> <div data-bbox="792 1034 1106 1214" style="border: 1px solid black; padding: 5px;"> $\begin{aligned} \eta\mu(\pi - \omega) &= \eta\mu\omega \\ \sigma\upsilon\nu(\pi - \omega) &= -\sigma\upsilon\nu\omega \\ \epsilon\varphi(\pi - \omega) &= -\epsilon\varphi\omega \\ \sigma\varphi(\pi - \omega) &= -\sigma\varphi\omega \end{aligned}$ </div> <div data-bbox="1122 1034 1435 1214" style="border: 1px solid black; padding: 5px;"> $\begin{aligned} \eta\mu(\pi + \omega) &= -\eta\mu\omega \\ \sigma\upsilon\nu(\pi + \omega) &= -\sigma\upsilon\nu\omega \\ \epsilon\varphi(\pi + \omega) &= \epsilon\varphi\omega \\ \sigma\varphi(\pi + \omega) &= \sigma\varphi\omega \end{aligned}$ </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div data-bbox="792 1289 1106 1469" style="border: 1px solid black; padding: 5px;"> $\begin{aligned} \eta\mu(2\pi - \omega) &= -\eta\mu\omega \\ \sigma\upsilon\nu(2\pi - \omega) &= \sigma\upsilon\nu\omega \\ \epsilon\varphi(2\pi - \omega) &= -\epsilon\varphi\omega \\ \sigma\varphi(2\pi - \omega) &= -\sigma\varphi\omega \end{aligned}$ </div> <div data-bbox="1122 1289 1435 1469" style="border: 1px solid black; padding: 5px;"> $\begin{aligned} \eta\mu(-\omega) &= -\eta\mu\omega \\ \sigma\upsilon\nu(-\omega) &= \sigma\upsilon\nu\omega \\ \epsilon\varphi(-\omega) &= -\epsilon\varphi\omega \\ \sigma\varphi(-\omega) &= -\sigma\varphi\omega \end{aligned}$ </div> </div>	 <div style="display: flex; justify-content: space-around;"> <div data-bbox="1480 970 1794 1230" style="border: 1px solid black; padding: 5px;"> $\begin{aligned} \eta\mu\left(\frac{\pi}{2} - \omega\right) &= \sigma\upsilon\nu\omega \\ \sigma\upsilon\nu\left(\frac{\pi}{2} - \omega\right) &= \eta\mu\omega \\ \epsilon\varphi\left(\frac{\pi}{2} - \omega\right) &= \sigma\varphi\omega \\ \sigma\varphi\left(\frac{\pi}{2} - \omega\right) &= \epsilon\varphi\omega \end{aligned}$ </div> <div data-bbox="1809 970 2123 1230" style="border: 1px solid black; padding: 5px;"> $\begin{aligned} \eta\mu\left(\frac{\pi}{2} + \omega\right) &= \sigma\upsilon\nu\omega \\ \sigma\upsilon\nu\left(\frac{\pi}{2} + \omega\right) &= -\eta\mu\omega \\ \epsilon\varphi\left(\frac{\pi}{2} + \omega\right) &= -\sigma\varphi\omega \\ \sigma\varphi\left(\frac{\pi}{2} + \omega\right) &= -\epsilon\varphi\omega \end{aligned}$ </div> </div> <div style="display: flex; justify-content: space-around; margin-top: 20px;"> <div data-bbox="1480 1246 1794 1506" style="border: 1px solid black; padding: 5px;"> $\begin{aligned} \eta\mu\left(\frac{3\pi}{2} - \omega\right) &= -\sigma\upsilon\nu\omega \\ \sigma\upsilon\nu\left(\frac{3\pi}{2} - \omega\right) &= -\eta\mu\omega \\ \epsilon\varphi\left(\frac{3\pi}{2} - \omega\right) &= \sigma\varphi\omega \\ \sigma\varphi\left(\frac{3\pi}{2} - \omega\right) &= \epsilon\varphi\omega \end{aligned}$ </div> <div data-bbox="1809 1246 2123 1506" style="border: 1px solid black; padding: 5px;"> $\begin{aligned} \eta\mu\left(\frac{3\pi}{2} + \omega\right) &= -\sigma\upsilon\nu\omega \\ \sigma\upsilon\nu\left(\frac{3\pi}{2} + \omega\right) &= \eta\mu\omega \\ \epsilon\varphi\left(\frac{3\pi}{2} + \omega\right) &= -\sigma\varphi\omega \\ \sigma\varphi\left(\frac{3\pi}{2} + \omega\right) &= -\epsilon\varphi\omega \end{aligned}$ </div> </div>

Τριγωνομετρικές Συναρτήσεις

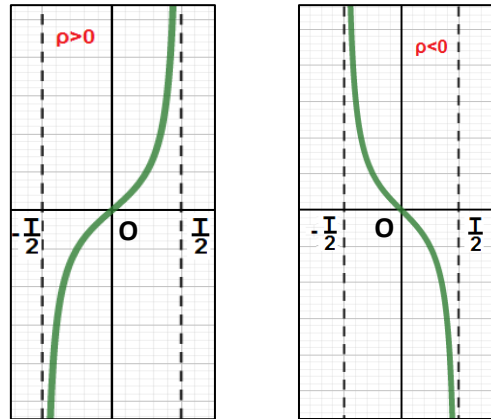
$$f(x) = \rho \cdot \eta\mu(\omega x)$$

- $\max f(x) = |\rho|$
- $\min f(x) = -|\rho|$
- $T = \frac{2\pi}{\omega}$



$$f(x) = \rho \cdot \epsilon\varphi(\omega x)$$

- $T = \frac{\pi}{\omega}$

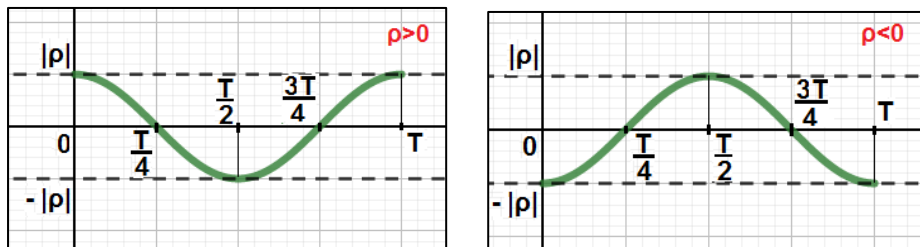


Τριγωνομετρ. αριθμοί αθροίσματος γωνιών

$$\begin{aligned} \eta\mu(\alpha + \beta) &= \eta\mu\alpha \sigma\upsilon\nu\beta + \sigma\upsilon\nu\alpha \eta\mu\beta \\ \eta\mu(\alpha - \beta) &= \eta\mu\alpha \sigma\upsilon\nu\beta - \sigma\upsilon\nu\alpha \eta\mu\beta \\ \sigma\upsilon\nu(\alpha - \beta) &= \sigma\upsilon\nu\alpha \sigma\upsilon\nu\beta + \eta\mu\alpha \eta\mu\beta \\ \sigma\upsilon\nu(\alpha + \beta) &= \sigma\upsilon\nu\alpha \sigma\upsilon\nu\beta - \eta\mu\alpha \eta\mu\beta \\ \epsilon\varphi(\alpha + \beta) &= \frac{\epsilon\varphi\alpha + \epsilon\varphi\beta}{1 - \epsilon\varphi\alpha \epsilon\varphi\beta} \\ \epsilon\varphi(\alpha - \beta) &= \frac{\epsilon\varphi\alpha - \epsilon\varphi\beta}{1 + \epsilon\varphi\alpha \epsilon\varphi\beta} \\ \sigma\varphi(\alpha + \beta) &= \frac{\sigma\varphi\alpha \sigma\varphi\beta - 1}{\sigma\varphi\alpha + \sigma\varphi\beta} \\ \sigma\varphi(\alpha - \beta) &= \frac{\sigma\varphi\alpha \sigma\varphi\beta + 1}{\sigma\varphi\beta - \sigma\varphi\alpha} \end{aligned}$$

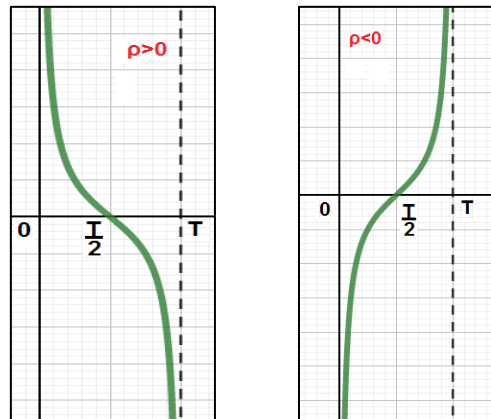
$$f(x) = \rho \cdot \sigma\upsilon\nu(\omega x)$$

- $\max f(x) = |\rho|$
- $\min f(x) = -|\rho|$
- $T = \frac{2\pi}{\omega}$



$$f(x) = \rho \cdot \sigma\varphi(\omega x)$$

- $T = \frac{\pi}{\omega}$



Τριγωνομετρ. αριθμοί γωνίας 2α - Τύποι υποβιβασμού

$$\begin{aligned} \eta\mu 2\alpha &= 2\eta\mu\alpha \sigma\upsilon\nu\alpha \\ \sigma\upsilon\nu 2\alpha &= \begin{cases} \sigma\upsilon\nu^2\alpha - \eta\mu^2\alpha \\ 1 - 2\eta\mu^2\alpha \\ 2\sigma\upsilon\nu^2\alpha - 1 \end{cases} \\ \epsilon\varphi 2\alpha &= \frac{2\epsilon\varphi\alpha}{1 - \epsilon\varphi^2\alpha} \\ \sigma\varphi 2\alpha &= \frac{\sigma\varphi^2\alpha - 1}{2\sigma\varphi\alpha} \end{aligned}$$

Τύποι αποτετραγωνισμού - Διπλασιασμού

$$\begin{aligned} \eta\mu^2\alpha &= \frac{1 - \sigma\upsilon\nu 2\alpha}{2} \\ \sigma\upsilon\nu^2\alpha &= \frac{1 + \sigma\upsilon\nu 2\alpha}{2} \\ \epsilon\varphi 2\alpha &= \frac{1 - \sigma\upsilon\nu 2\alpha}{1 + \sigma\upsilon\nu 2\alpha} \end{aligned}$$

Τριγωνομετρικές Εξισώσεις – Τύποι λύσεων

<p>Η εξίσωση $\eta\mu\chi = \alpha$</p> <ul style="list-style-type: none"> • Αν $\alpha < -1$ ή $\alpha > 1$ τότε η εξίσωση είναι αδύνατη • Αν $-1 \leq \alpha \leq 1$ τότε $\eta\mu\chi = \alpha \Leftrightarrow \eta\mu\chi = \eta\mu\theta \Leftrightarrow \begin{cases} \chi = 2κπ + \theta \\ \text{ή} \\ \chi = 2κπ + \pi - \theta \end{cases}, κ \in \mathbb{Z}$	<p>Η εξίσωση $\sigma\upsilon\nu\chi = \alpha$</p> <ul style="list-style-type: none"> • Αν $\alpha < -1$ ή $\alpha > 1$ τότε η εξίσωση είναι αδύνατη • Αν $-1 \leq \alpha \leq 1$ τότε $\sigma\upsilon\nu\chi = \alpha \Leftrightarrow \sigma\upsilon\nu\chi = \sigma\upsilon\nu\theta \Leftrightarrow \begin{cases} \chi = 2κπ + \theta \\ \text{ή} \\ \chi = 2κπ - \theta \end{cases}, κ \in \mathbb{Z}$	<p>Οι εξισώσεις $\epsilon\phi\chi = \alpha$ και $\sigma\phi\chi = \alpha$</p> <p>$\epsilon\phi\chi = \alpha \Leftrightarrow \epsilon\phi\chi = \epsilon\phi\theta \Leftrightarrow \chi = κπ + \theta, κ \in \mathbb{Z}$</p> <p>$\sigma\phi\chi = \alpha \Leftrightarrow \sigma\phi\chi = \sigma\phi\theta \Leftrightarrow \chi = κπ + \theta, κ \in \mathbb{Z}$</p>
--	--	--

Τριγωνομετρικός Πίνακας

Γωνία ω	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
ω	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\eta\mu\omega$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\sigma\upsilon\nu\omega$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\epsilon\phi\omega$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	----	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	----	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\sigma\phi\omega$	----	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	----	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	----