

ΤΥΠΟΛΟΓΙΟ ΜΑΘΗΜΑΤΙΚΩΝ Α΄ ΛΥΚΕΙΟΥ**ΔΥΝΑΜΕΙΣ**

$$\alpha^v = \underbrace{\alpha \cdot \alpha \cdots \alpha}_{v\text{-παράγοντες}}, \alpha^0 = 1, \alpha \neq 0, \alpha^1 = \alpha$$

$$\alpha^\mu \cdot \alpha^v = \alpha^{\mu+v}, \frac{\alpha^\mu}{\alpha^v} = \alpha^{\mu-v}, (\alpha^\mu)^v = \alpha^{\mu \cdot v}$$

$$(\alpha \cdot \beta)^v = \alpha^v \cdot \beta^v, \left(\frac{\alpha}{\beta}\right)^v = \frac{\alpha^v}{\beta^v}, \alpha^{-v} = \frac{1}{\alpha^v}, \alpha \neq 0$$

$$\left(\frac{\alpha}{\beta}\right)^{-v} = \left(\frac{\beta}{\alpha}\right)^v, \alpha, \beta \neq 0$$

ΤΑΥΟΤΗΤΕΣ

$$(\alpha \pm \beta)^2 = \alpha^2 \pm 2\alpha\beta + \beta^2, (\alpha + \beta)(\alpha - \beta) = \alpha^2 - \beta^2$$

$$(\alpha \pm \beta)^3 = \alpha^3 \pm 3\alpha^2\beta + 3\alpha\beta^2 \pm \beta^3, \alpha^3 \pm \beta^3 = (\alpha \pm \beta)(\alpha^2 \mp \alpha\beta + \beta^2)$$

$$(\alpha + \beta + \gamma)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2\alpha\beta + 2\beta\gamma + 2\alpha\gamma$$

$$\alpha^v - \beta^v = (\alpha - \beta)(\alpha^{v-1} + \alpha^{v-2}\beta + \cdots + \alpha\beta^{v-2} + \beta^{v-1}), v \text{ φυσικός}$$

$$\alpha^v + \beta^v = (\alpha + \beta)(\alpha^{v-1} - \alpha^{v-2}\beta + \cdots - \alpha\beta^{v-2} + \beta^{v-1}), v \text{ περιττός}$$

ΑΠΟΛΥΤΗ ΤΙΜΗ

$$|\alpha| = \begin{cases} \alpha, \alpha \geq 0 \\ -\alpha, \alpha < 0 \end{cases}, |\alpha| \geq 0, -\alpha \leq |\alpha| \leq \alpha, |\alpha|^2 = \alpha^2, |\alpha| = |-\alpha|$$

$$\text{Αν } \theta \geq 0 \text{ τότε ισχύει } |x| = \theta \Leftrightarrow x = \theta \text{ η } x = -\theta$$

$$|x| = |\theta| \Leftrightarrow x = \theta \text{ η } x = -\theta, \sqrt{\alpha^2} = |\alpha|$$

$$|x| \leq \theta \Leftrightarrow -\theta \leq x \leq \theta, \theta \geq 0$$

$$|x| \geq \theta \Leftrightarrow x \geq \theta \text{ η } x \leq -\theta, \theta \geq 0$$

$$|\alpha \cdot \beta| = |\alpha| \cdot |\beta|, \left|\frac{\alpha}{\beta}\right| = \frac{|\alpha|}{|\beta|}, \beta \neq 0, \left||\alpha| - |\beta|\right| \leq |\alpha + \beta| \leq |\alpha| + |\beta|$$

ΡΙΖΕΣ

$$\sqrt[v]{\alpha} = x \Leftrightarrow x^v = \alpha, \text{ όπου } \alpha \geq 0, x \geq 0, v \in \mathbb{N}$$

$$(\sqrt[v]{\alpha})^v = \sqrt[v]{\alpha^v} = \alpha, \alpha \geq 0, \sqrt{\alpha^2} = |\alpha| \text{ όπου } \alpha \in \mathbb{R}$$

$$\text{Αν } \alpha, \beta > 0: \sqrt[v]{\alpha} \cdot \sqrt[v]{\beta} = \sqrt[v]{\alpha\beta}, \frac{\sqrt[v]{\alpha}}{\sqrt[v]{\beta}} = \sqrt[v]{\frac{\alpha}{\beta}}, \sqrt[v]{\sqrt[\mu]{\alpha}} = \sqrt[\mu v]{\alpha}$$

$$(\sqrt[v]{\alpha})^k = \sqrt[v]{\alpha^k}, \sqrt[v]{\alpha^k} = \sqrt[kv]{\alpha^{kp}}, \sqrt[v]{\alpha^k} = \alpha^{\frac{k}{v}}, \sqrt[v]{\alpha^v \beta} = \alpha \sqrt[v]{\beta}$$

$$x^v = \alpha \Rightarrow \begin{cases} v: \text{αρτιος} \rightarrow x = \pm \sqrt[v]{\alpha}, \text{οταν } : \alpha \geq 0, \text{αδυνατη, οταν } : \alpha < 0 \\ v: \text{περιττος} \rightarrow x = \sqrt[v]{\alpha}, \text{οταν } : \alpha \geq 0, x = -\sqrt[v]{-\alpha}, \text{οταν } : \alpha < 0 \end{cases}$$

$$x^v = \alpha^v \Rightarrow \begin{cases} \alpha v, v: \text{αρτιος, τοτε} \rightarrow x = \pm \alpha \\ \alpha v, v: \text{περιττος, τοτε} \rightarrow x = \alpha \end{cases}$$

ΠΛΗΘΟΣ ΡΙΖΩΝ ΤΗΣ ΕΞΙΣΩΣΗΣ $\alpha x^2 + \beta x + \gamma = 0, \alpha \neq 0$

ΔΙΑΚΡΙΝΟΥΣΑ : $\Delta = \beta^2 - 4\alpha\gamma$	
Av $\Delta > 0$	Έχει δύο ρίζες άνισες $x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha}$
Av $\Delta = 0$	Έχει μια διπλή ρίζα $x_1 = x_2 = \frac{-\beta}{2\alpha}$
Av $\Delta < 0$	Δεν έχει πραγματικές ρίζες

ΤΥΠΟΙ ΤΟΥ Vieta

Av x_1, x_2 ρίζες της εξίσωσης $\alpha x^2 + \beta x + \gamma = 0, \alpha \neq 0$, τότε
$S = x_1 + x_2 = -\frac{\beta}{\alpha}$, $P = x_1 \cdot x_2 = \frac{\gamma}{\alpha}$, $x^2 - Sx + P = 0$

ΠΡΟΣΗΜΟ ΤΗΣ $f(x) = \alpha x^2 + \beta x + \gamma, \alpha \neq 0, x \in \mathbb{R}$

Av $\Delta > 0$	$-\infty$	x_1	x_2	$+\infty$
$F(x)$	Ομόσημο του α	Ετερόσημο του α	Ομόσημο του α	
Av $\Delta = 0$	$-\infty$	$-\beta/2\alpha$		$+\infty$
$F(x)$	Ομόσημο του α	Ομόσημο του α		
Av $\Delta < 0$	$-\infty$			$+\infty$
$F(x)$	Ομόσημο του α			

ΣΥΣΤΗΜΑ 2x2

$\begin{cases} \alpha x + \beta y = \gamma \\ \alpha' x + \beta' y = \gamma' \end{cases}$	$D = \begin{vmatrix} \alpha & \beta \\ \alpha' & \beta' \end{vmatrix}$, $D_x = \begin{vmatrix} \gamma & \beta \\ \gamma' & \beta' \end{vmatrix}$, $D_y = \begin{vmatrix} \alpha & \gamma \\ \alpha' & \gamma' \end{vmatrix}$
Av $D \neq 0$ έχει μοναδική λύση	$x = \frac{D_x}{D}$, $y = \frac{D_y}{D}$
Av $D = 0$:	$\begin{cases} D_x \neq 0, D_y \neq 0 \rightarrow \text{ΑΔΥΝΑΤΟ} \\ D_x = D_y = 0, \begin{cases} \text{ΑΟΡΙΣΤΟ} \\ \text{ΑΔΥΝΑΤΟ, αν } \alpha = \alpha' = \beta = \beta' = 0, \text{ και } \gamma \neq 0, \eta, \gamma' \neq 0 \end{cases} \end{cases}$

ΤΡΙΓΩΝΟΜΕΤΡΙΑ

$\eta\mu^2\theta + \sigma\upsilon\nu^2\theta = 1$, $\epsilon\phi\theta = \frac{\eta\mu\theta}{\sigma\upsilon\nu\theta}$, $\sigma\phi\theta = \frac{\sigma\upsilon\nu\theta}{\eta\mu\theta}$, $\epsilon\phi\theta \cdot \sigma\phi\theta = 1$
$\eta\mu(-x) = -\eta\mu(x)$, $\eta\mu(2\kappa\pi + x) = \eta\mu x$, $\eta\mu(\pi \pm x) = \mp \eta\mu x$, $\eta\mu\left(\frac{\pi}{2} \pm x\right) = \sigma\upsilon\nu x$
$\sigma\upsilon\nu(-x) = \sigma\upsilon\nu x$, $\sigma\upsilon\nu(2\kappa\pi + x) = \sigma\upsilon\nu x$, $\sigma\upsilon\nu(\pi \pm x) = -\sigma\upsilon\nu x$, $\sigma\upsilon\nu\left(\frac{\pi}{2} \pm x\right) = \mp \eta\mu x$
$\epsilon\phi(-x) = -\epsilon\phi x$, $\epsilon\phi(2\kappa\pi + x) = \epsilon\phi x$, $\epsilon\phi(\pi \pm x) = \pm \epsilon\phi x$, $\epsilon\phi\left(\frac{\pi}{2} \pm x\right) = \mp \sigma\phi x$
$\sigma\phi(-x) = -\sigma\phi x$, $\sigma\phi(2\kappa\pi + x) = \sigma\phi x$, $\sigma\phi(\pi \pm x) = \pm \sigma\phi x$, $\sigma\phi\left(\frac{\pi}{2} \pm x\right) = \mp \epsilon\phi x$