Διαγώνισμα *Γ Τάξης Ενιαίου Λυκείου*

Απλή Αρμονική Ταλάντωση - Κρούσεις

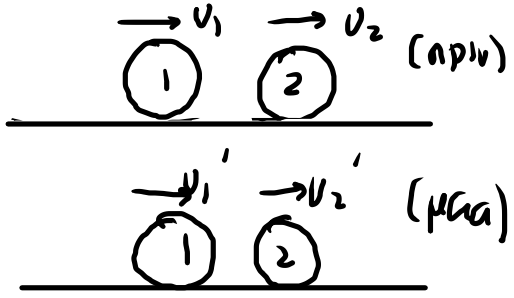
Σύνολο Σελίδων: δέκα (10) - Διάρκεια Εξέτασης: 3 ώρες

Σάββατο 23 Ιουλίου 2022

Θέμα Α Α.1 → (γ) / Α.2 → (β) / Α.3 → (β) / Α.4 → δ / Α.5 → Σ, Λ, Λ, Λ, Σ

Θέμα Β.1

↳ (α)



$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2 \quad (1)$$

$$v_2' = \frac{m_2 - m_1}{m_1 + m_2} v_2 + \frac{2m_1}{m_1 + m_2} \cdot v_1 \quad (2)$$

$v_1 = 4v_2 \longrightarrow \frac{v_1'}{v_2'} = \frac{1}{4}$

$$(1) \rightarrow u_1' = \frac{4m_1 - 4m_2 + 2m_2}{m_1 + m_2} u_2 \Rightarrow u_1' = \frac{4m_1 - 2m_2}{m_1 + m_2} u_2$$

$$(2) \rightarrow u_2' = \frac{m_2 - m_1 + 8m_1}{m_1 + m_2} u_2 \Rightarrow u_2' = \frac{m_2 + 7m_1}{m_1 + m_2} u_2$$

$$\frac{u_1'}{u_2'} = \frac{4m_1 - 2m_2}{m_2 + 7m_1} = \frac{1}{4} \Rightarrow 16m_1 - 8m_2 = m_2 + 7m_1 \Rightarrow$$

$$9m_1 = 9m_2 \Rightarrow \underline{\underline{m_1 = m_2}}$$

Ή Θα μπορούσαμε να σκεφτούμε!

$$u_1 + u_1' = u_2 + u_2' \Rightarrow u_1 + \frac{u_2'}{4} = \frac{u_1}{4} + u_2'$$

$$\Rightarrow \frac{3u_1}{4} = \frac{3u_2'}{4} \Rightarrow \underline{\underline{u_2' = u_1}} \longrightarrow \text{Ανταλλαγή ταχυτήτων} \rightarrow m_1 = m_2$$

Θέμα Β.2

↳ (γ)

$$|F| = \frac{|F_{max}|}{4} \Rightarrow |-Dx| = \frac{|DA|}{4} \Rightarrow x = \frac{A}{4}$$

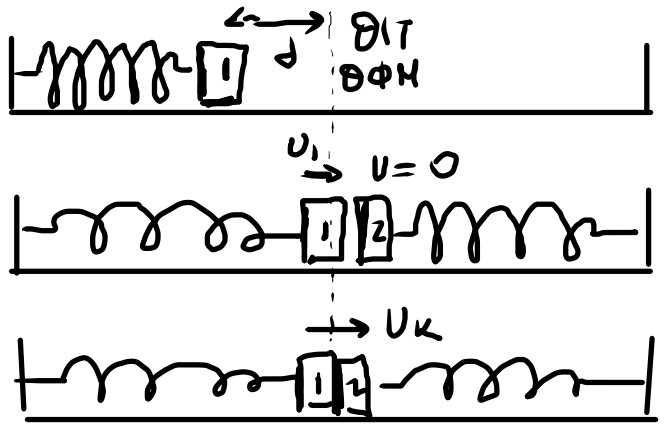
$$U = \frac{1}{2}Dx^2 = \frac{1}{2}D\frac{A^2}{16} \Rightarrow U = \frac{E}{16}$$

$$\frac{K}{U} = 15$$

$$\text{ΑΔΕΤ}: E = K + U \Rightarrow K = E - \frac{E}{16} \Rightarrow K = \frac{15E}{16}$$

Θέμα Β.3

↳ (β)



Για το Σ_1:

$$A_1 = d \quad , \quad v_1 = v_{max} = \omega A_1$$

$$D = K = m \cdot \omega^2 \Rightarrow \omega = \sqrt{\frac{K}{m}}$$

$$v_K = \omega' A_2 \quad , \quad D' = 2K = (m+m)\omega'^2$$

$$2K = 2m\omega'^2$$

$$\omega' = \sqrt{\frac{2K}{2m}} = \omega$$

Α.Δ.Ο. $\vec{p}_{\Omega}^{\,\text{πριν}} = \vec{p}_{\Omega}^{\,\text{μετά}} \Rightarrow m \cdot v_1 = (m+m)v_K$

$$\Rightarrow v_1 = 2v_K \Rightarrow \omega A_1 = 2\omega A_2 \Rightarrow A_1 = 2A_2$$

Θέμα Γ

$$K = 2 - 50x^2 \ (SI)$$

Στην αα2 : $E = K + U$

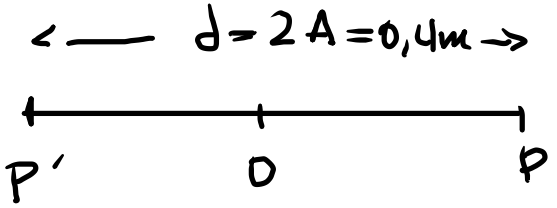
Άρα $K = E - U = E - \frac{1}{2}Dx^2$

Γ.1

Άρα $E = 2\,J$ και $\frac{1}{2}D = 50 \Rightarrow D = 100\,N/m$

$E = \frac{1}{2}DA^2 \Rightarrow A = \sqrt{\frac{2E}{D}} \Rightarrow A = 0,2m$ και $D = m\omega^2 \Rightarrow \omega = \sqrt{\frac{D}{m}} \Rightarrow \omega = 10\,r/s$

και $T = \frac{2\pi}{\omega} \Rightarrow T = \frac{\pi}{5}\,s$

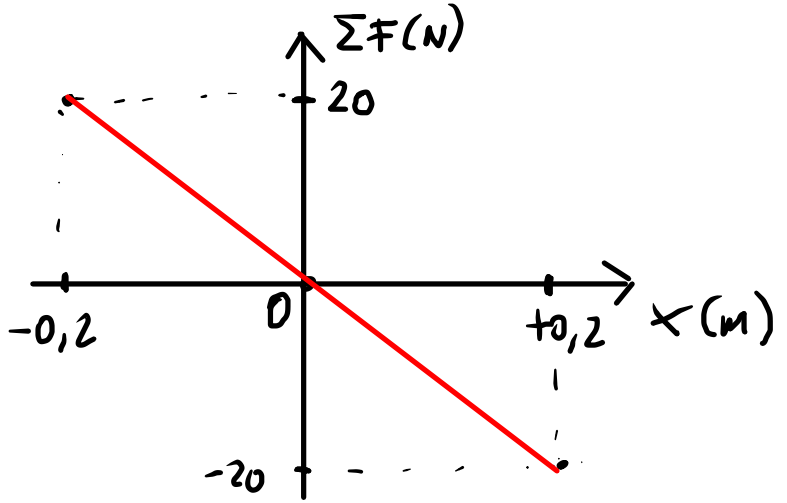


και $\Delta t = \frac{T}{2} = \frac{\pi}{10}\,s$

Γ.2

$$\Sigma F = -Dx$$

Άρα $\begin{bmatrix} \Sigma F = -100x \ (SI) \\ -0,2m \le x \le +0,2m \end{bmatrix}$



Γ.3 $x = A\eta\mu(\omega t + \varphi_0) \mid t_0 = 0 \rightarrow x = A = A\eta\mu(0 + \varphi_0)$

$\eta\mu\varphi_0 = 1 \Rightarrow \varphi_0 = \frac{\pi}{2}$

Αρα $x = 0,2\,\eta\mu\left(10t + \frac{\pi}{2}\right)$ (SI)

$x\left(\frac{2\pi}{5}\right) = 0,2\,\eta\mu\left(10\frac{2\pi}{5} + \frac{\pi}{2}\right) = +0,2m$

$\Delta x = x_{τελ} - x_{αρχ}$

$= 0,2 - 0,2$

$\Rightarrow \underline{\Delta x = 0}$

(*) η $t_1 = \frac{2\pi}{5} = 2T \rightarrow$ έχει επιστρέψει στην αρχική θέση

Και $S_{ολ} = 8A \Rightarrow \underline{S_{ολ} = 1,6m}$

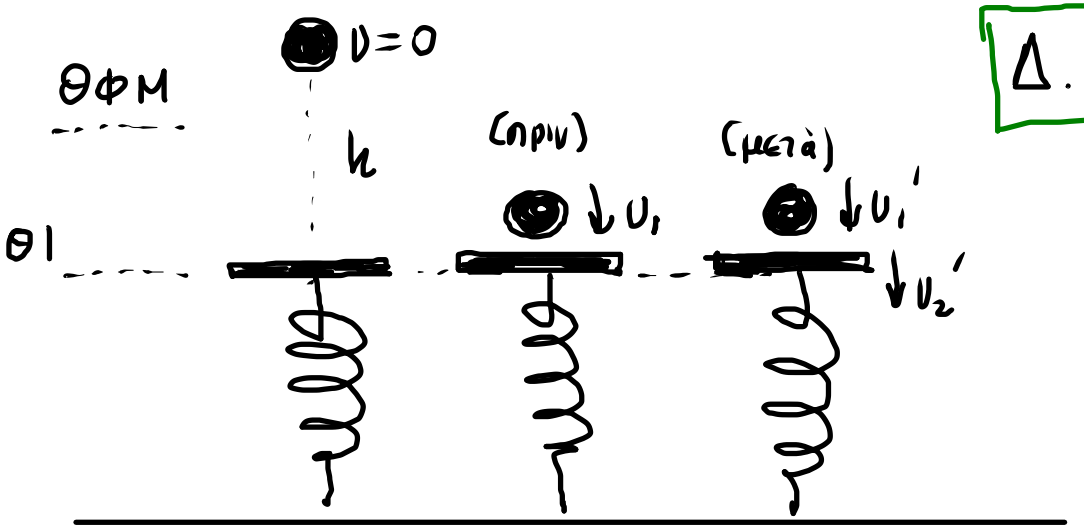
Γ.4 $t = \frac{T}{12} \rightarrow v = v_{max}\cdot\sigma\upsilon\nu\left(\frac{2\pi}{T}\cdot\frac{T}{12} + \frac{\pi}{2}\right) = \omega A\cdot\sigma\upsilon\nu\left(\frac{\pi}{6} + \frac{\pi}{2}\right)$

$v = 2\cdot\left(-\frac{1}{2}\right) \Rightarrow \underline{v = -1\,m/s}$

$K = \frac{1}{2}mv^2 = 0,5J$

$\frac{K}{E}\cdot 100\% = \frac{0,5}{2}\cdot 100\% = \underline{25\%}$

Θέμα Δ



Για την πτώση

$\Delta K = \Sigma W$

$\frac{1}{2} m u_1^2 = mgh$

$u_1 = \sqrt{2gh} \Rightarrow u_1 = 4\,m/s$

Δ.1

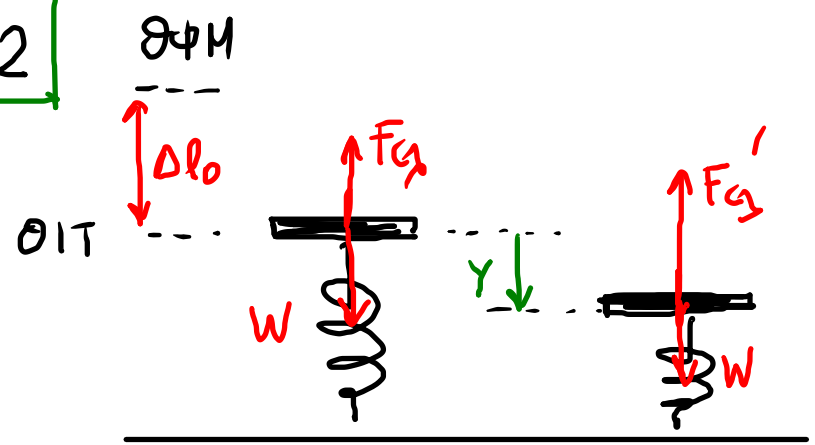
Για την κρούση

$u_1' = \frac{m-M}{m+M} u_1 \Rightarrow u_1' = -\frac{u_1}{4}$

$u_2' = \frac{2m}{m+M} u_1 \Rightarrow u_2' = \frac{3u_1}{4}$

Άρα $u_1' = -1\,m/s$ και $u_2' = 3\,m/s$

Δ.2



Στην ΘΙΤ: $\Sigma F = 0 \Rightarrow F_{ελ} = W \Rightarrow k\Delta l_0 = Mg$ (1)

Στην τυχαία θέση: $\Sigma F = W - F_{ελ}' = W - k(\Delta l_0 + y)$

$\Sigma F = W - k\Delta l_0 - ky \overset{(1)}{\Rightarrow} \Sigma F = -ky$

επομένως με $D = k$

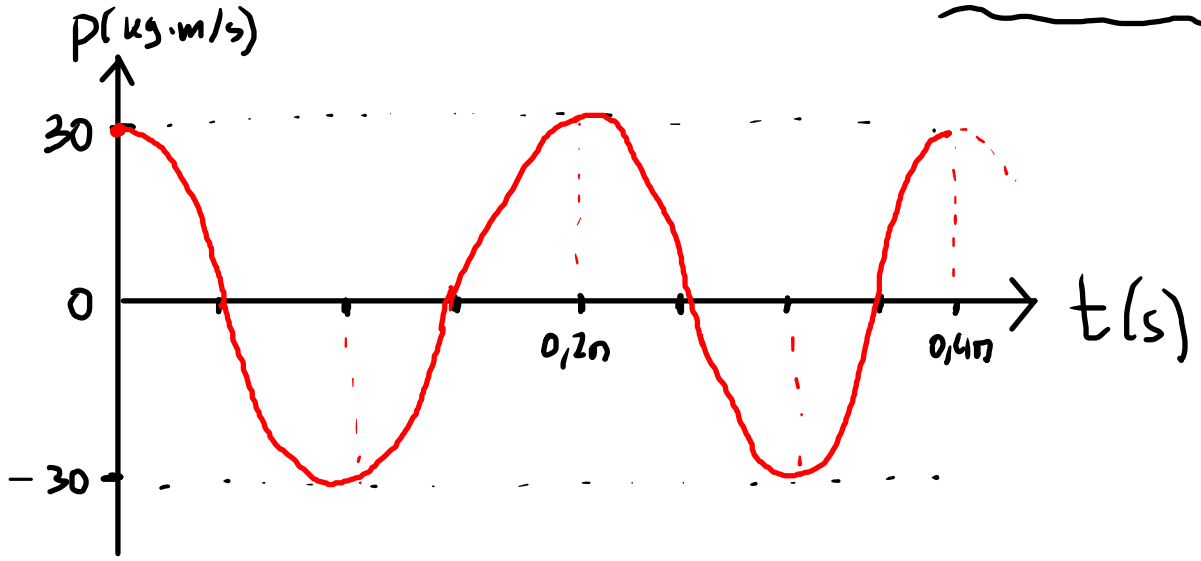
$D = M\omega^2 \Rightarrow \omega = \sqrt{\frac{k}{M}} \Rightarrow \underline{\omega = 10\,r/s}$ οπότε $\omega = 2\pi f \Rightarrow \underline{f = \frac{5}{\pi} Hz}$

Δ.3. $x = A\eta\mu(\omega t + \varphi_0)$, $U_2' = U_{max} = \omega A \Rightarrow \underline{A = 0{,}3m}$

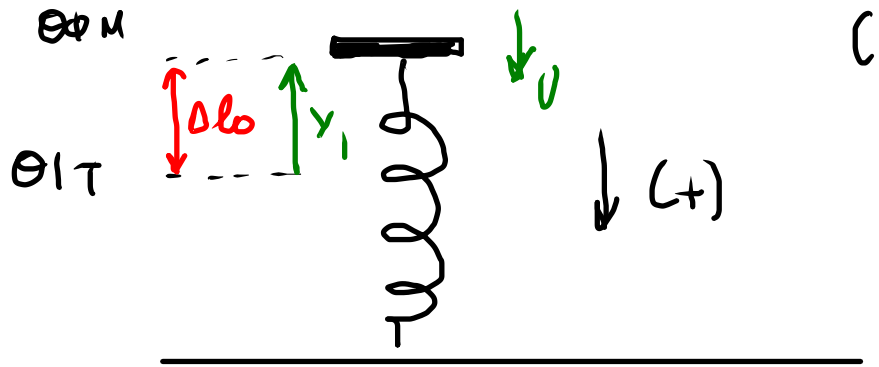
$t_0 = 0 \rightarrow \Theta\Gamma(x=0)$ $\left.\begin{array}{l} \searrow U > 0 \\ \end{array}\right\}$ $A\eta\mu\varphi_0 = 0 \rightarrow \varphi_0 = 0$ ή $\varphi_0 = \pi$ / $0 \le \varphi_0 \le 2\pi$

$U_{max}\sigma\upsilon\nu\varphi_0 > 0 \rightarrow \sigma\upsilon\nu\varphi_0 > 0 \rightarrow \underline{\varphi_0 = 0}$

$P = MU = MU_{max} \cdot \sigma\upsilon\nu(\omega t) \Rightarrow \underline{P = 30\sigma\upsilon\nu(10t)\ (SI)}$



Δ.4



(1) → $\Delta l_0 = \frac{Mg}{K} \Rightarrow \Delta l_0 = 0,1\,m$

Βρίσκεται σην $y_1 = -0,1\,m$ με $U > 0$

ΑΔΕΤ : $E = K + U \Rightarrow$

$$\frac{1}{2} D A^2 = \frac{1}{2} M U^2 + \frac{1}{2} D y_1^2$$

$$U^2 = \frac{K}{M}\left(A^2 - y_1^2\right) = 100\left(\frac{9}{100} - \frac{1}{100}\right)$$

$$U = +2\sqrt{2}\ m/s$$

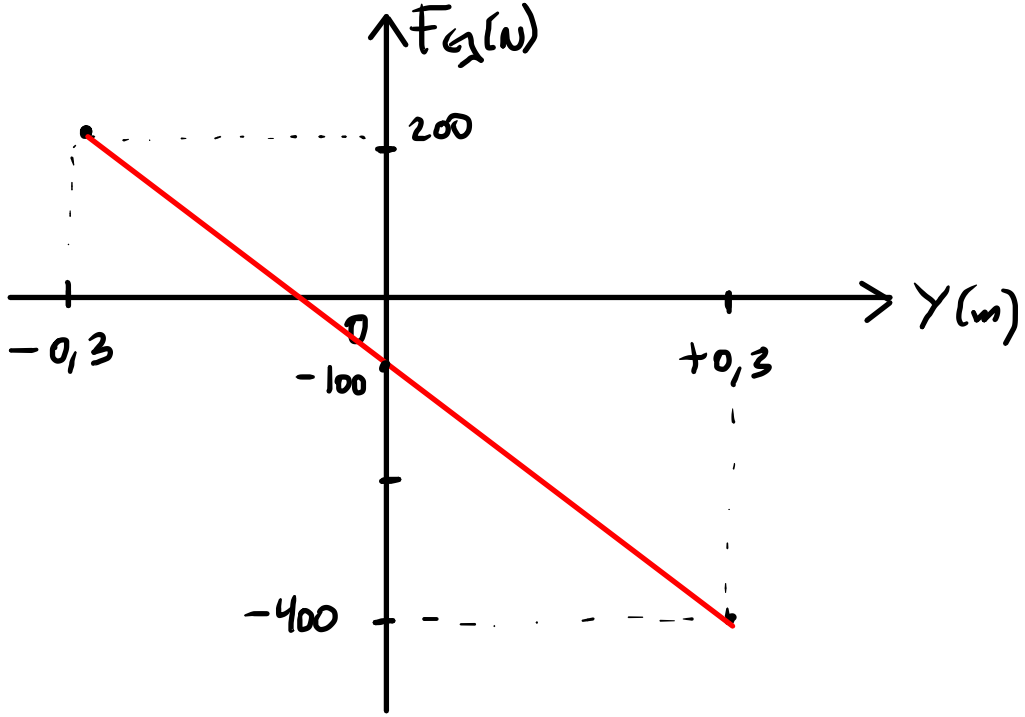
$$\frac{dK}{dt} = \Sigma F \cdot U = -D \cdot y_1 \cdot U$$

$$\frac{dK}{dt} = -1000 \cdot (-0,1) \cdot (+2\sqrt{2})$$

$$\frac{dK}{dt} = +200\sqrt{2}\ J/s$$

Δ.5 $\Sigma \vec{F} = -D\vec{y} \Rightarrow \vec{F}_{el} + \vec{W} = -D\vec{y} \Rightarrow F_{el} + W = -Dy$

$\Rightarrow F_{el} = -W - Dy \Rightarrow \boxed{\begin{array}{c} F_{el} = -100 - 1000\,y \quad (SI) \\ -0{,}3m \leq y \leq +0{,}3m \end{array}}$



Δ.6 $\dfrac{E}{V_{el(max)}} = \dfrac{\frac{1}{2}DA^2}{\frac{1}{2}K(\Delta l_{max})^2}$

$\dfrac{E}{V_{el(max)}} = \left(\dfrac{A}{\Delta l_{max}}\right)^2 = \left(\dfrac{A}{\Delta l_0 + A}\right)^2$

$\boxed{\dfrac{E}{V_{el(max)}} = \dfrac{9}{16}}$