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Μάθημα: Αγγλική Γλώσσα & Μαθηματικά

Μαθήτρια: Αθανασοπούλου Μαρία (Τάξη Β1)

Υπεύθυνοι Καθηγητές: Χριστοπούλου Μαρία, ΠΕ06 & Παρτσαδάκης Σπυρίδων, ΠΕ03

The Golden Ratio

Introduction

My name is Mary Athanasopoulou and I am in the second year of Experimental Senior-High School of Patras. My research concerns a mathematical analogy that can be found throughout nature, the one of “phi”. The topic is one that interests me a lot so I did some research on *Youtube* videos to get general knowledge on the topic and then I searched the subject more intensively in order to prepare this project. This paper contains the history of “phi” as well as the way to construct it from a mathematical point of view. Last but not least, there is a list of examples that show the importance of the golden ratio in nature, where it can be found and what the Fibonacci sequence has to do with the golden number 1.618.

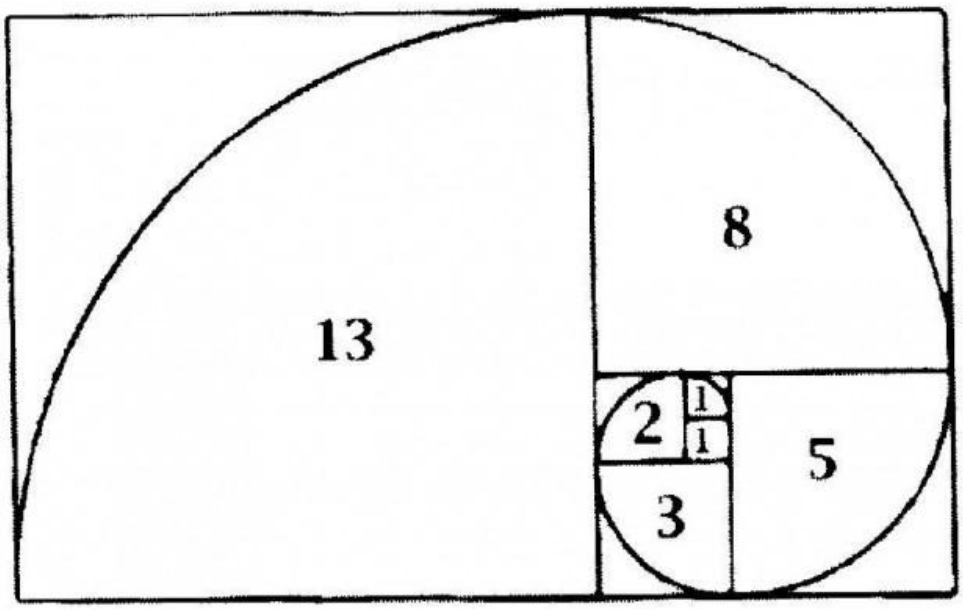
The Golden Ratio

The *Golden ratio* is a special number found by dividing a line into two parts so that the longer part divided by the smaller part is also equal to the whole length divided by the longer part. It is often symbolized using the «φ»=“**phi**” (the 21st letter of the Greek alphabet). In an equation form, it looks like this: a/b = (a+b)/a = 1.6180339887498948420 …

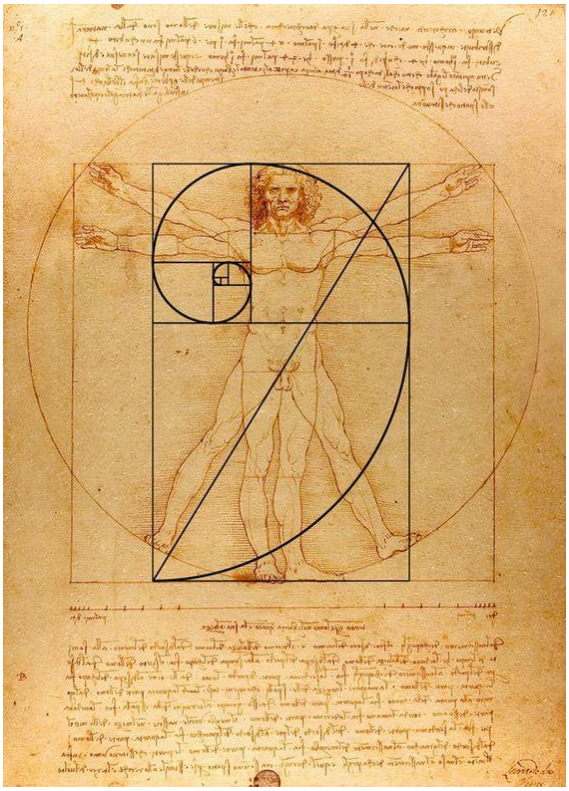
Phi is usually rounded off to **1.618**. This number has been discovered and rediscovered many times, which is why it has so many names: the *Golden mean*, the *Golden section*, *divine proportion*, etc. Historically, the number can be seen in the architecture of many ancient creations, like the Great Pyramids and the Parthenon. In the Great Pyramid of Giza, the length of each side of the base is 756 feet with a height of 481 feet. The ratio of the base to the height is roughly 1.5717, which is close to the Golden ratio.

Phidias (500 B.C. - 432 B.C.) was a Greek sculptor and mathematician who is thought to have applied phi to the design of sculptures for the Parthenon. Plato (428 B.C. - 347 B.C.) considered the Golden ratio to be the most universally binding of mathematical relationships. Later, Euclid (365 B.C. - 300 B.C.) linked the Golden ratio to the construction of a pentagram.

Around 1200, mathematician **Leonardo Fibonacci** discovered the unique properties of the *Fibonacci sequence*. This sequence ties directly into the Golden ratio because if you take any two successive Fibonacci numbers, their ratio is very close to the Golden ratio. As the numbers get higher, the ratio becomes even closer to 1.618. For example, the ratio of 3 to 5 is 1.666. But the ratio of 13 to 21 is 1.625. Getting even higher, the ratio of 144 to 233 is 1.618. These numbers are all successive numbers in the Fibonacci sequence.



In 1509, Luca Pacioli wrote a book that refers to the number as the "Divine Proportion," which was illustrated by **Leonardo da Vinci**. Da Vinci later called this *sectio aurea* or the Golden section. The Golden ratio was used to achieve balance and beauty in many Renaissance paintings and sculptures. Da Vinci himself used the Golden ratio to define all of the proportions in his *Last Supper*, including the dimensions of the table and the proportions of the walls and backgrounds. The Golden ratio also appears in da Vinci's *Vitruvian Man* and the *Mona Lisa*. Other artists who employed the Golden ratio include **Michelangelo**, **Raphael**, **Rembrandt**, **Seurat**, and **Salvador Dali**.



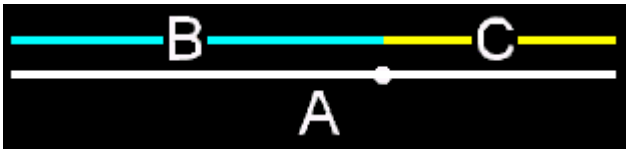
The term "phi" was coined by American mathematician Mark Barr in the 1900s. Phi has continued to appear in mathematics and physics, including the 1970s Penrose Tiles, which allowed surfaces to be tiled in five-fold symmetry. In the 1980s, phi appeared in quasi crystals, a then-newly discovered form of matter.

Construction of phi

Phi, like Pi, is a ratio defined by a geometric construction.

Just as pi (p) is the ratio of the circumference of a circle to its diameter, phi is simply the ratio of the line segments that result when a line is divided in one very special and unique way.

Divide a line so that:



…the ratio of the length of the entire line (A) to the length of larger line segment (B) is the same as the ratio of the length of the larger line segment (B) to the length of the smaller line segment (C).

This happens only at the point where:

A is 1.618 … times B and B is 1.618 … times C.

Alternatively, C is 0.618… of B and B is 0.618… of A.

Phi with an upper case "P" is 1.618 0339 887 …, while phi with a lower case "p" is 0.6180339887, the reciprocal of Phi and also Phi minus 1.

What makes phi even more unusual is that it can be derived in many ways and shows up in relationships throughout the universe.

Phi in our lives

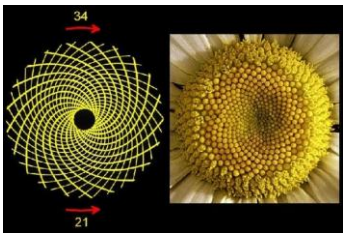
Phi is more than an obscure term found in mathematics and physics. It appears around us in our daily lives, even in our aesthetic views. Studies have shown that when test subjects view random faces, the ones they deem most attractive are those with solid parallels to the Golden ratio. Faces judged as the most attractive show Golden ratio proportions between the width of the face and the width of the eyes, nose, and eyebrows. The test subjects weren't mathematicians or physicists familiar with phi — they were just average people, and the Golden ratio elicited an instinctual reaction.

The Golden ratio also appears in all forms of nature and science. Some unexpected places include:

Flower petals: The number of petals on some flowers follows the Fibonacci sequence. It is believed that in the Darwinian processes, each petal is placed to allow for the best possible exposure to sunlight and other factors.

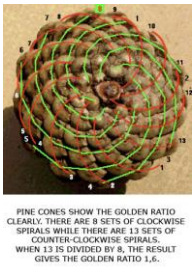



Seed heads: The seeds of a flower are often produced at the center and migrate outward to fill the space. For example, sunflowers follow this pattern.

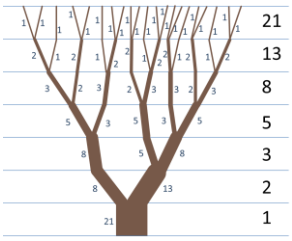


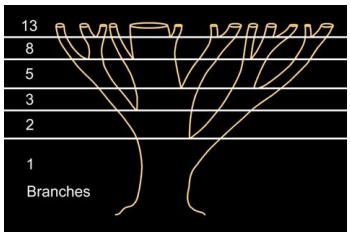

Pinecones: The spiral pattern of the seed pods spiral upward in opposite directions. The number of steps the spirals take tend to match Fibonacci numbers.



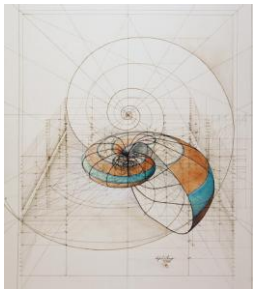
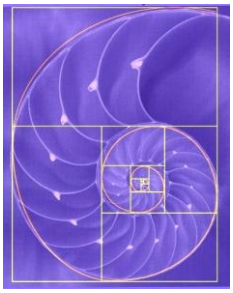
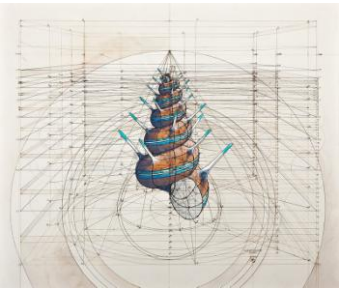
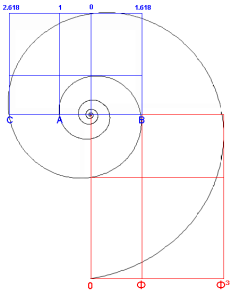


Tree branches: The way tree branches form or split is an example of the Fibonacci sequence. Root systems and algae exhibit this formation pattern.

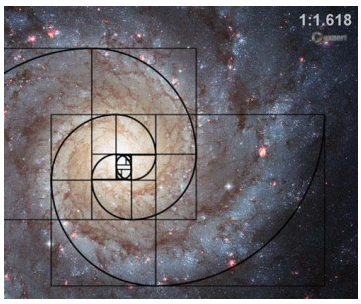


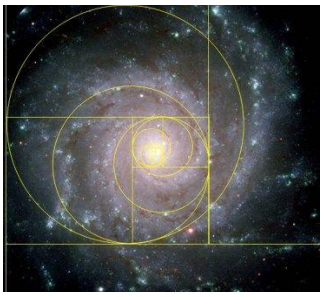



Shells: Many shells, including snail shells and nautilus shells, are perfect examples of the Golden spiral.

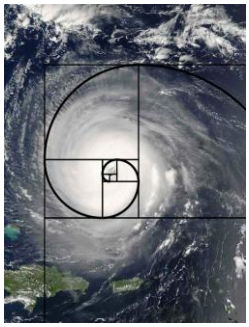
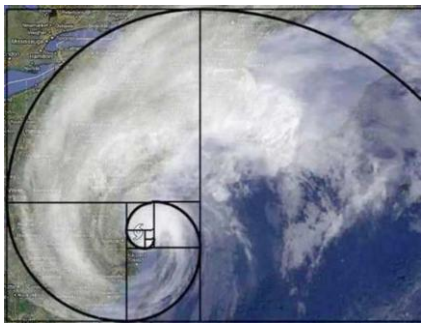
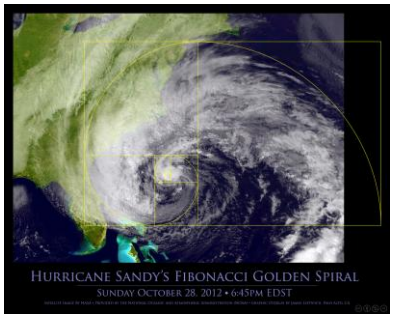





Spiral galaxies: The Milky Way has a number of spiral arms, each of which has a logarithmic spiral of roughly 12 degrees. The shape of the spiral is identical to the Golden spiral, and the Golden rectangle can be drawn over any spiral galaxy.

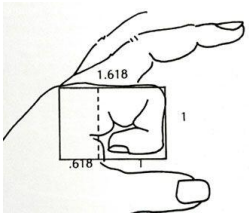


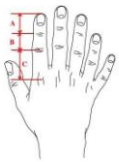
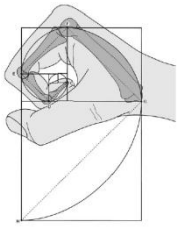
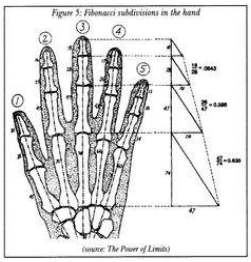



Hurricanes: Much like shells, hurricanes often display the Golden spiral.

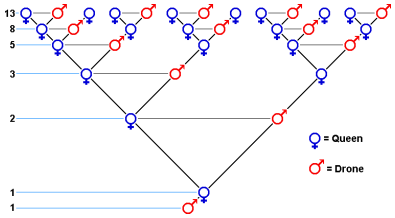




Fingers: The length of our fingers, each section from the tip of the base to the wrist is larger than the preceding one by roughly the ratio of phi.

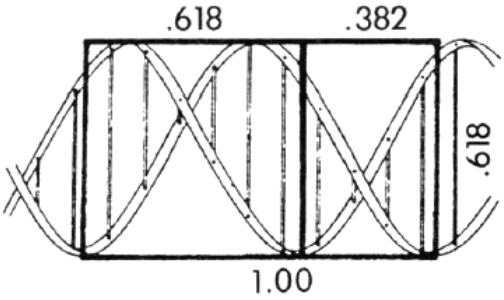


Animal bodies: The measurement of the human navel to the floor and the top of the head to the navel is the Golden ratio. But we are not the only examples of the Golden ratio in the animal kingdom; dolphins, starfish, sand dollars, sea urchins, ants and honeybees also exhibit the proportion.

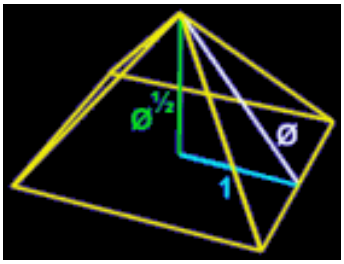



DNA molecules: A DNA molecule measures 34 angstroms by 21 angstroms at each full cycle of the double helix spiral. In the Fibonacci series, 34 and 21 are successive numbers.



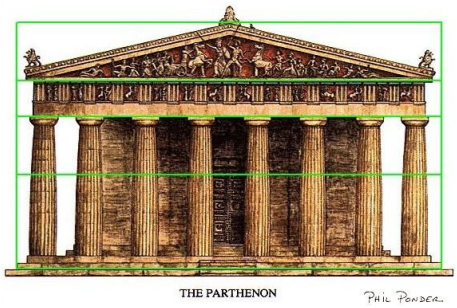
Phi (Φ) the Golden Section, has been used by mankind for centuries in architecture

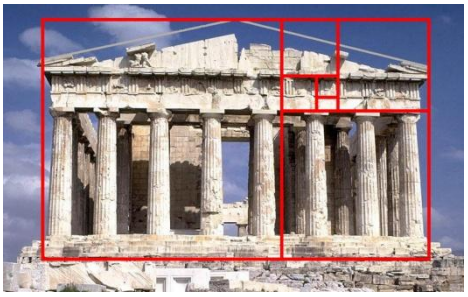
Its use started perhaps as early as with the Egyptians in the design of the pyramids. When the basic phi relationships are used to create a right triangle, the dimensions of the great pyramids of Egypt are formed, with the geometry shown below creating an angle of 51.83 degrees, the cosine of which is phi, or 0.618.



The Parthenon

The ancient Greek Euclid ((365–300 BC) wrote about the golden ratio in his work "Elements" as "dividing a line in the extreme and mean ratio." The Parthenon, built in 447 to 438 BC, appears to use it in some aspects of its design to achieve beauty and balance. The illustration below shows one of the ways that the golden ratio is often reported to appear in its design. This, however, is subject to some debate, as the application of the golden ratio is often not accurately described in many sources. Furthermore, using the second step of the Parthenon seems somewhat arbitrary. There are, however, other dimensions of the Parthenon which appear to be golden ratios.

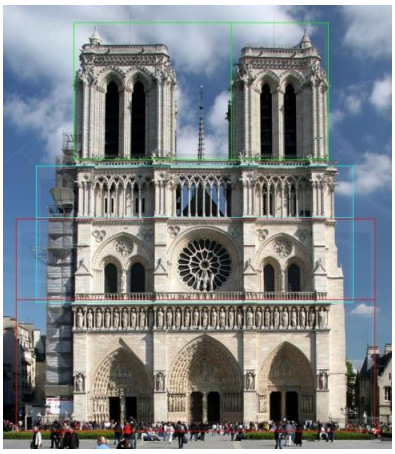




Notre Dame

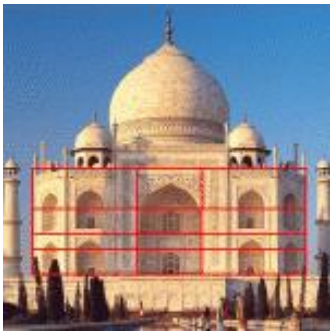
Notre Dame in Paris, which was built in between 1163 and 1250, appears to have golden ratio proportions in a number of its key proportions of design. Although it is rather asymmetrical in its design and difficult to measure photographically because of parallax distortions, the golden ratio lines of the green, blue and red rectangles conform closely to the major architectural lines, which represent:

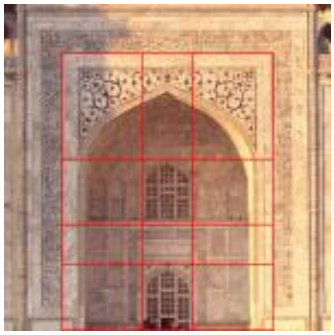
- Red – Vertical height of base at ground level: Top of first level: Top of second floor
- Blue – Vertical height of base of second level: Top of second level: Top of third level
- Green – Horizontal width of outside of left top section: Inside of top right section: Outside of top right section



The Taj Mahal

Renaissance artists of the 1500's in the time of Leonardo Da Vinci knew it as the Divine Proportion. In India, it was used in the construction of the Taj Mahal, which was completed in 1648.





The United-Nations Secretariat Building

The United Nations building also reflects the golden ratio in a number of aspects of its design.

UN Secretariat window configuration reveals golden proportions

Toronto's Canadian National (CN) Tower

The CN Tower in Toronto, the tallest tower and freestanding structure in the world, contains the golden ratio in its design. The ratio of the observation deck at 342 meters to the total height of 553.33 is 0.618 or phi, the reciprocal of Phi!

Conclusion

Phi is a mathematical analogy that is of great importance in our lives even if we do not realize it. What is most awe-inspiring is the fact that nature has such harmony and every inch of it is not randomly put together. Nature is mysterious is some ways and we will not seize trying to comprehend its wonders.

Bibliography

https://www.goldennumber.net/what-is-phi/

http://www.livescience.com/37704-phi-golden-ratio.html

https://www.goldennumber.net/leonardo-da-vinci-golden-ratio-art/

https://en.wikipedia.org/wiki/Phi