

Θέμα A

A<sub>1</sub>, A<sub>2</sub> A<sub>3</sub> Θρησκιά δυο γινότων βιβλίων

A<sub>4</sub> α) Σ β) Σ γ) Λ δ) Λ ε) Σ

Θέμα B

B<sub>1</sub>

$$|z-3|^2 + |z+3|^2 = 36 \Rightarrow$$

$$(z-3)(\bar{z}-3) + (z+3)(\bar{z}+3) = 36 \Rightarrow$$

$$\cancel{z\bar{z}} - \cancel{3\bar{z}} - \cancel{3\bar{z}} + 9 + \cancel{z\bar{z}} + \cancel{3\bar{z}} + \cancel{3\bar{z}} + 9 = 36 \Rightarrow$$

$$2z\bar{z} = 18 \Rightarrow z\bar{z} = 9 \Rightarrow |z|^2 = 9 \Rightarrow |z| = 3$$

Κύριος κύριος  
(0, 0) από την 3.

B<sub>2</sub>

$$|z_1 - z_2| = 3\sqrt{2} \Rightarrow |z_1 - z_2|^2 = 18 \Rightarrow (z_1 - z_2)(\bar{z}_1 - \bar{z}_2) = 18 \Rightarrow$$

$$z_1\bar{z}_1 - z_1\bar{z}_2 - z_2\bar{z}_1 + z_2\bar{z}_2 = 18 \Rightarrow \boxed{z_1\bar{z}_2 + z_2\bar{z}_1 = 0} \quad ①$$

$$\text{Είναι } |z_1 + z_2|^2 = (z_1 + z_2)(\bar{z}_1 + \bar{z}_2) = z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2 \stackrel{①}{=} \\ = 9 + 0 + 9 = 18$$

$$\text{Άρα } |z_1 + z_2|^2 = 18 \Rightarrow |z_1 + z_2| = \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}.$$

B<sub>3</sub>

$$|2w-1| = |w-2| \Rightarrow |2w-1|^2 = |w-2|^2 \Rightarrow (2w-1)(2\bar{w}-1) = (w-2)(\bar{w}-2) \Rightarrow$$

$$4w\bar{w} - 2w - 2\bar{w} + 1 = w\bar{w} - 2w - 2\bar{w} + 4 \Rightarrow$$

$$3w\bar{w} = 4-1 \Rightarrow w\bar{w} = 1 \Rightarrow |w|^2 = 1 \Rightarrow |w| = 1$$

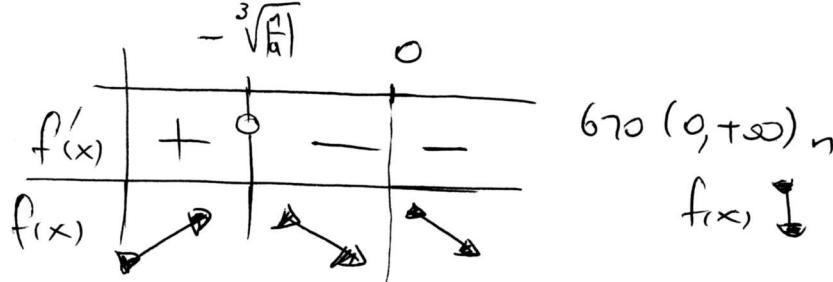
Κύριος κύριος (0, 0) ρ = 1.

OTMA Γ]

$$\boxed{f(x) = \frac{2}{x} + ax^2 + b, x > 0}$$

$$f'(x) = -\frac{2}{x^2} + 2ax = \frac{2ax^3 - 2}{x^2} = 2 \cdot \frac{ax^3 - 1}{x^2}, \quad f'(x) = 0 \Rightarrow ax^3 - 1 = 0 \Rightarrow$$

$$x^3 = \frac{1}{a} \Rightarrow x = -\sqrt[3]{\left| \frac{1}{a} \right|}$$



Ex] If  $f(x) \rightarrow 0$  as  $x \rightarrow \infty$  then  $\lim_{x \rightarrow \infty} f(x)$

$\lim_{x \rightarrow 0^+} f(x) = +\infty$  αριθμόνα τον οποίον φέρεται  $f_1$  μετά  $f_1 > 0$

$\lim_{x \rightarrow +\infty} f(x) = -\infty$ , apă numărătoare  $\rightarrow +\infty$  și  $f(x) \rightarrow -\infty$  pentru  $x \in \mathbb{R}$ .  
 În plus,  $f(7_2) < 0$

Aao S. Bolzanorumprin öre

W67+  $f(x_0) = 0$  i.e.  $\exists x_0 \in \mathbb{R} \setminus \{x_{107}\}$   $(0, +\infty)$

Paradiso. ० प्रस शो (०, +०) n f(x) और अंदर से

$$\boxed{3} \quad \lim_{x \rightarrow 0^+} f(x) = +\infty$$

f(x)  $\stackrel{\text{unstetig}}{=}$  an  $a \in \mathbb{R}$ ,  $b \in \mathbb{R}$  an  $x = 0$  einm

ΕΛΛΑΣΚΟΠΥΨΗ ΑΣΥΜΜΤΗΣΗ ή  $f(x)$ ,  $f_n(x)$ ,  $f_m(x)$

O atoms YY'

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{2+ax^3+bx}{x} = \lim_{x \rightarrow +\infty} \frac{x^3 \left( a + \frac{b}{x^2} + \frac{2}{x^3} \right)}{x} = \lim_{x \rightarrow +\infty} x^2 \left( a + \frac{b}{x^2} + \frac{2}{x^3} \right)$$

$$= (+\infty) \cdot a = \begin{cases} +\infty, & a > 0 \\ \text{anp. Wegen } a \neq 0 \\ -\infty, & a < 0 \end{cases}$$

Tia  $a=0$   $\lim_{x \rightarrow \infty} f(x) = 0$ ,  
 Apa  $0 < x' < \lim_{x \rightarrow +\infty} f(x)$

$\Gamma_4$

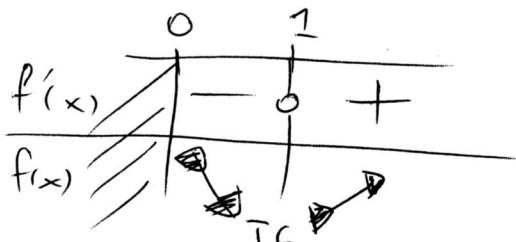
Ano θεωρηγια Fermat  $f'(1) = 0 \Rightarrow 2a - 2 = 0 \Rightarrow$   
 $a = 1.$

$$f(x) = \frac{2}{x} + x^2 + b \quad \text{και} \quad f(x_0) = 7 \quad \text{και} \quad f(1) = 7 \quad \text{αριθμ}$$

$$2 + 1 + b = 7 \Rightarrow b = 4$$

$$f(x) = \frac{2}{x} + x^2 + 4, \quad f'(x) = \frac{-2}{x^2} + 2x = \frac{2x^3 - 2}{x^2} = \frac{2(x^3 - 1)}{x^2}$$

$$f'(x) = 0 \Rightarrow x^3 - 1 = 0 \Rightarrow x = 1$$



To  $x_0 = 1$  ειναι T.f.

Θεμα Δ

Δ₁

Η  $f(x)$  διαχωριστηγια με  $f(x)$  συνεχης στο  $\mathbb{R}$ .  
 $\lim_{x \rightarrow 0} \frac{f(x) + n \nu x}{x^2 - x} = 2$ , Ειναι  $\lim_{x \rightarrow 0} \frac{f(x) + n \nu x}{x^2 - x} = \frac{f(0) + 0}{0} = \frac{f(0)}{0}$

Να προσεις  $f(0) = 0$  ηταξι αλλιως  $\lim_{x \rightarrow 0} \frac{f(x) + n \nu x}{x^2 - x} \neq$  ηπαριστημενης αριθμ.

Επομη  $\lim_{x \rightarrow 0} \frac{f(x) + n \nu x}{x(x-1)} = \lim_{x \rightarrow 0} \left( \frac{f(x)}{x(x-1)} + \frac{n \nu x}{x(x-1)} \right)$  κατια τοιμης,  
 $\delta$ ρια να αποχων αριθ.

$$= \lim_{x \rightarrow 0} \left( \frac{f(x) - f(0)}{x} \cdot \frac{1}{x-1} \right) + \lim_{x \rightarrow 0} \left( \frac{n \nu x}{x} \cdot \frac{1}{x-1} \right) = f'(0) \cdot (-1) + 1 \cdot (-1)$$

$$= -f'(0) - 1$$

Ημ αριθμ:  $2 = -f'(0) - 1 \Rightarrow$

$$\boxed{f'(0) = -3}$$

Επομη  $\boxed{f(1) = f'(0) = -3}$

$\Delta_2$

$g(x)$  και θ. Rolle αρ  
 $\frac{\text{προτίμως να } 16x^4 \text{ είναι.}}{g(0)=g(1)}$

$$a = -3 + 4a$$

$$-3a = -3$$

$$\boxed{a=1}$$

$g(x) \in C^1[0, 1]$

παραγωγής  $f'(0, 1)$

$$g(0) = f(0) + a = a$$

$$g(1) = f(1) + 4a = -3 + 4a$$

$\Delta_3$

H  $g(x)$  kavonoiει zis npouadis, τω θ. Rolle 670 [0, 1]

και  $\exists \exists f(0, 1)$  w67t  $g'(7) = 0 \Rightarrow$   
 Ta<sub>λ</sub> kaxi67ou

$$f'(7) + 2(7+1) = 0 \Rightarrow f'(7) = -2(7+1)$$

$$\text{και } g'(x) = f'(x) + 2 \cdot (x+1)$$

$$g''(x) = f''(x) + 2 > 0 \Rightarrow n g'(x) \uparrow 670 [0, 1]$$

(πάνω σε  $g(x)$  είναι uprī 670 [0, 1])

$\Delta_4$  το  $\exists \in (0, 1)$  αρ 670 διαβητών

670 διαβητών

$0 < x < 7$  εξω:  $\left( \begin{array}{l} \text{μια} \\ \text{και} \end{array} \right)$  με  $g'(x) \uparrow$

$$g'(x) < g'(7) \Rightarrow g'(x) < 0 \quad 670 [0, 7]$$

$$\tau o \quad 0 < 7 < 1$$

Xupi) εται 670 δυο

σιαστηματα

$0 < x < 7$  και

$7 < x < 1$

με 670  $7 < x < 1 \Rightarrow g'(7) < g'(x) \Rightarrow$

$$g'(x) > 0$$

$$n g'(x) \uparrow 670 [7, 1]$$

Aρu70  $(7, g(7))$  τ. ε in,

$g(x)$ .