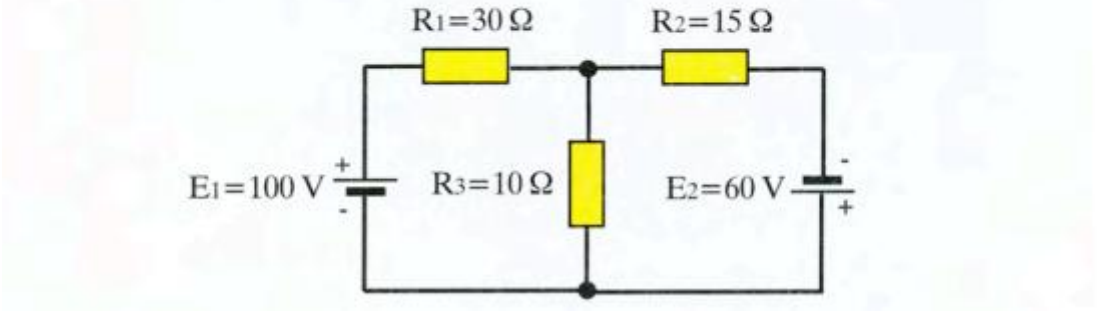
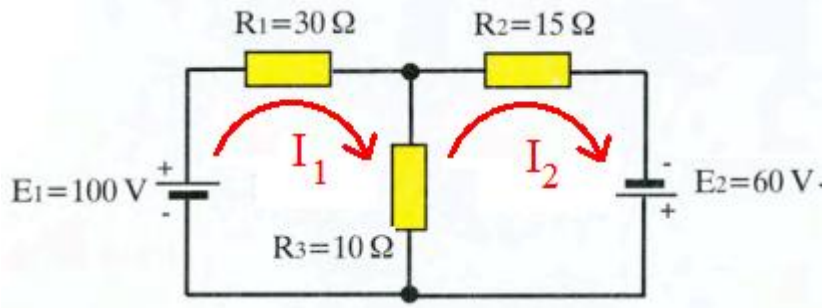
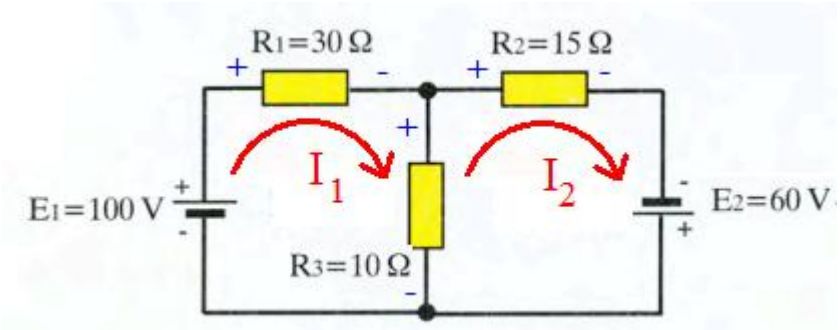
8. Στο κύκλωμα του σχήματος να υπολογιστεί το ρεύμα που διέρχεται από την αντίσταση R_3



Λύση

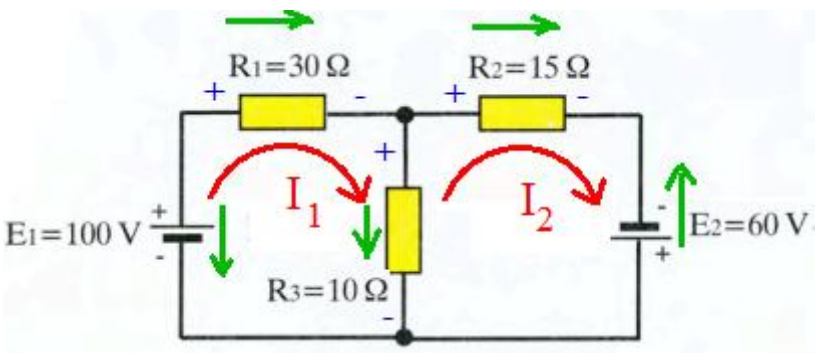
1ο)Μέθοδος των βρόγχων





β’ κανόνας του Kirchhoff σε κάθε βρόγχο:

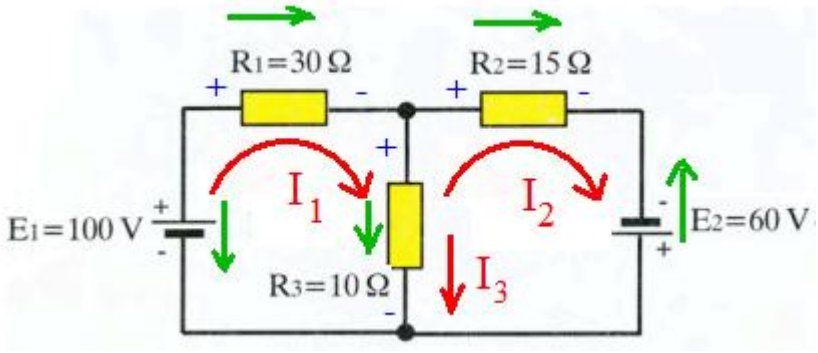
$$\left.\begin{array}{l} 1^{ος}\,\beta\rho\acute{o}\gamma\chi o\varsigma\colon E_1 + U_1 + U_3 = 0 \\ 2^{ος}\,\beta\rho\acute{o}\gamma\chi o\varsigma\colon E_2 + U_2 + U_3 = 0 \end{array}\right\} \Rightarrow$$



$$\left.\begin{matrix} 1^{ος}\,\beta\rho\acute{o}\gamma\chi o\varsigma\colon -E_1+U_1+U_3=0 \\ 2^{ος}\,\beta\rho\acute{o}\gamma\chi o\varsigma\colon -E_2+U_2-U_3=0 \end{matrix}\right\}\Rightarrow \left.\begin{matrix} -E_1+(R_1I_1)+(R_3I_3)=0 \\ -E_2+(R_2I_2)-(R_3I_3)=0 \end{matrix}\right\}\Rightarrow$$

$$\begin{matrix} -E_1+R_1I_1+R_3I_3=0 \\ -E_2+R_2I_2-R_3I_3=0 \end{matrix}$$

α’ κανόνας του Kirchhoff στον μοναδικό κόμβο:



$$I_1+I_2+I_3=0\Rightarrow I_1-I_2+I_3=0\Rightarrow I_3=I_1-I_2$$

I_1,I_3 :εισερχόμενα στον κόμβο

I_2: εξερχόμενο στον κόμβο

$$\left.\begin{matrix} -E_1+R_1I_1+R_3I_3=0 \\ -E_2+R_2I_2-R_3I_3=0 \\ I_3=I_1-I_2 \end{matrix}\right\}\Rightarrow \left.\begin{matrix} -E_1+R_1I_1+R_3(I_1-I_2)=0 \\ -E_2+R_2I_2-R_3(I_1-I_2)=0 \end{matrix}\right\}\Rightarrow$$

$$\left.\begin{matrix} -E_1+R_1I_1+R_3I_1-R_3I_2=0 \\ -E_2+R_2I_2-R_3I_1+R_3I_2=0 \end{matrix}\right\}\Rightarrow \left.\begin{matrix} -E_1+(R_1+R_3)I_1-R_3I_2=0 \\ -E_2-R_3I_1+(R_2+R_3)I_2=0 \end{matrix}\right\}\Rightarrow$$

$$\left.\begin{matrix} -100+(30+10)I_1-10I_2=0 \\ -60-10I_1+(15+10)I_2=0 \end{matrix}\right\}\Rightarrow$$

$$\left.\begin{matrix} \mathbf{-100+40I_1-10I_2=0} \\ \mathbf{-60-10I_1+25I_2=0} \end{matrix}\right\}\Rightarrow$$

$$\left.\begin{matrix} I_1=\dfrac{100+10I_2}{40}=\mathbf{2,5+0,25I_2} \\ -60-10(\mathbf{2,5+0,25I_2})+25I_2=0 \end{matrix}\right\}\Rightarrow$$

$$\left.\begin{matrix} I_1=2{,}5+0{,}25I_2 \\ -60-25-2{,}5I_2+25I_2=0 \end{matrix}\right\}\Rightarrow$$

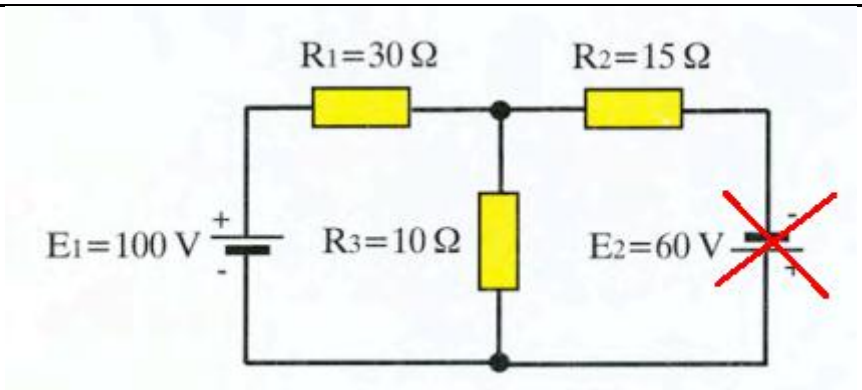
$$\left.\begin{matrix} I_1=2{,}5+0{,}25I_2 \\ -85+22{,}5I_2=0 \end{matrix}\right\}\Rightarrow$$

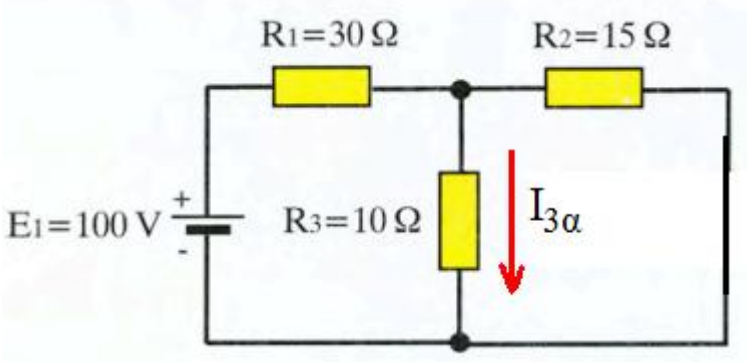
$$\begin{cases} I_1=2{,}5+0{,}25I_2=2{,}5+0{,}25\left(\mathbf{\dfrac{85}{22,5}}\right)=3{,}444A \\ I_2=\mathbf{\dfrac{85}{22,5}}=3.777A \end{cases}$$

Άρα:

$$I_3 = I_1 - I_2 = 3{,}44 - 3{,}777 = -0{,}33A$$

2°) Μέθοδος της επαλληλίας

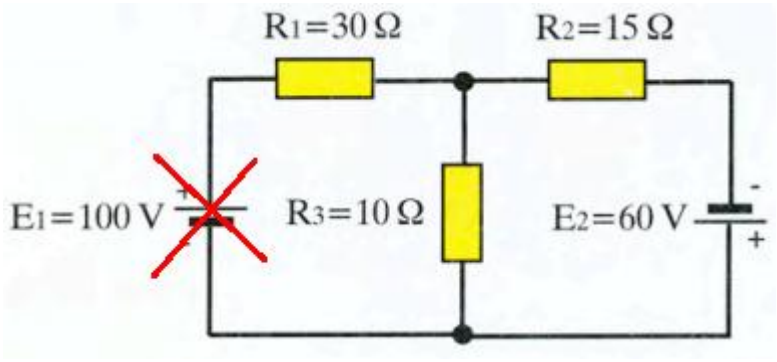


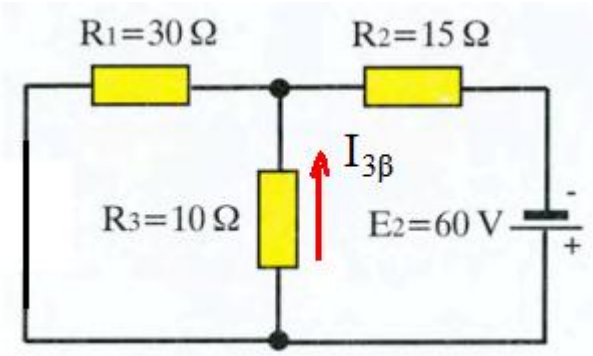


$$R_{o\lambda 1} = \frac{R_2 * R_3}{R_2 + R_3} + R_1 = \frac{15 * 10}{15 + 10} + 30 = 36\Omega$$

$$I_{o\lambda 1} = \frac{E_1}{R_{o\lambda 1}} = \frac{100}{36} = 2{,}777A$$

$$I_{3\alpha} = \frac{R_2}{R_3 + R_2} I_{o\lambda 1} = \frac{15}{10 + 15} 2{,}777 = \mathbf{1{,}666}A$$





$$R_{o\lambda 2} = \frac{R_1 * R_3}{R_1 + R_3} + R_2 = \frac{30 * 10}{30 + 10} + 15 = 22{,}5\Omega$$

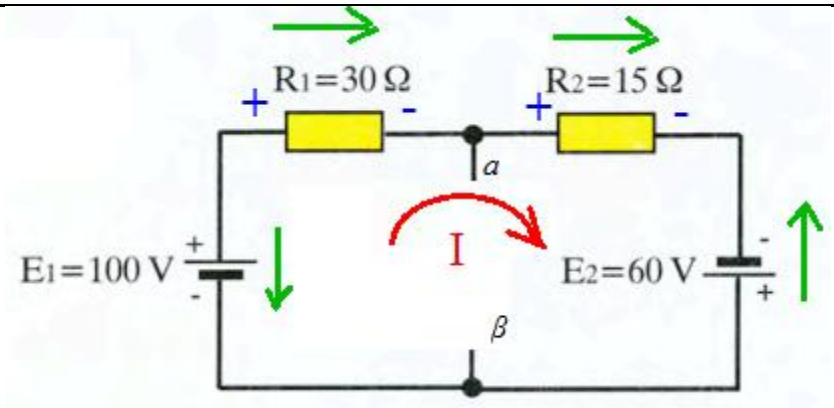
$$I_{o\lambda 2} = \frac{E_2}{R_{o\lambda 2}} = \frac{60}{22{,}5} = 2{,}666A$$

$$I_{3\beta} = \frac{R_1}{R_1 + R_3} I_{o\lambda 1} = \frac{30}{30 + 10} 2{,}666 = \mathbf{1{,}999}A$$

Άρα:

$$I_3 = I_{3\alpha} - I_{3\beta} = 1{,}66 - 1{,}99 = -0{,}33A$$

3°) Με το θεώρημα Thevenin



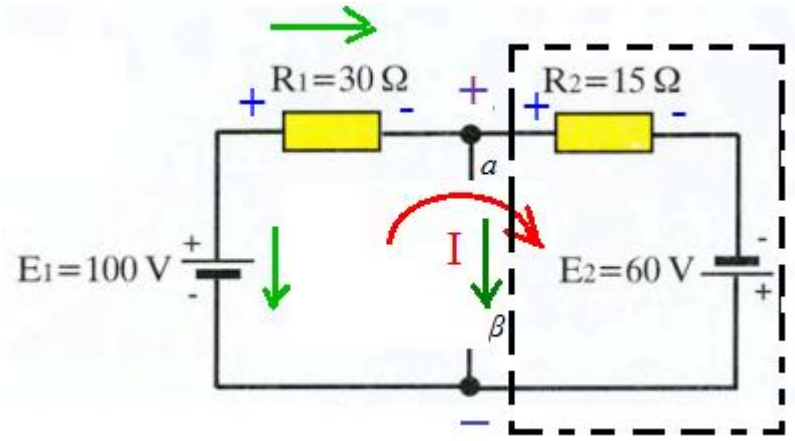
Υπολογιαμός της ισοδύναμης τάσης Thevenin

β' κανόνας του Kirchhoff στον βρόγχο:

$$-E_1 - E_2 + R_1 I + R_2 I = 0 \Leftrightarrow -E_1 - E_2 + (R_1 + R_2) I = 0 \Leftrightarrow$$

$$-100 - 60 + (30 + 15) I = 0 \Leftrightarrow -160 + 45I = 0 \Rightarrow$$

$$I = \frac{160}{45} = 3{,}555A$$

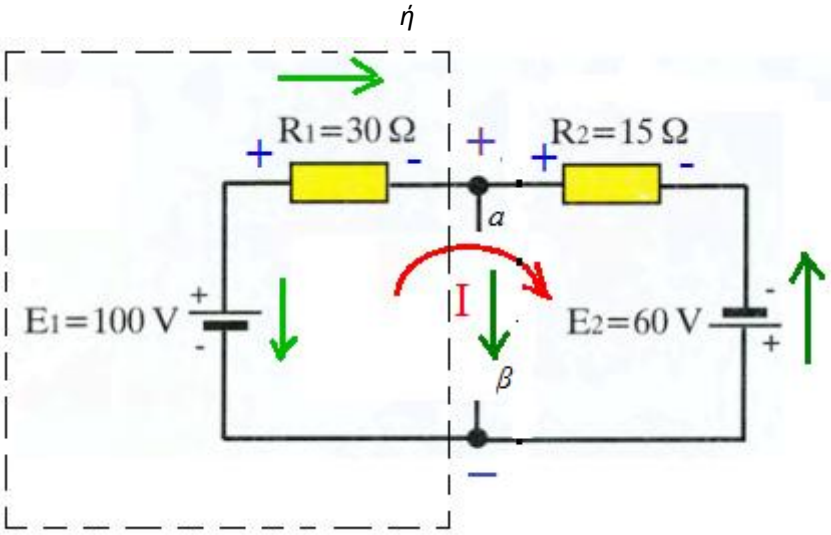


β' κανόνας του Kirchhoff στον βρόγχο:

$$-E_1 + R_1 I + U_{\alpha\beta} = 0 \Rightarrow$$

$$U_{\alpha\beta} = E_1 - R_1 I = 100 - 30 * 3{,}555 = -6{,}65V$$

ή

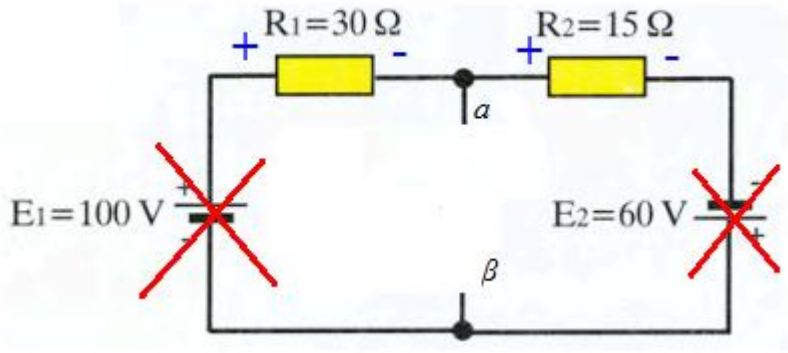


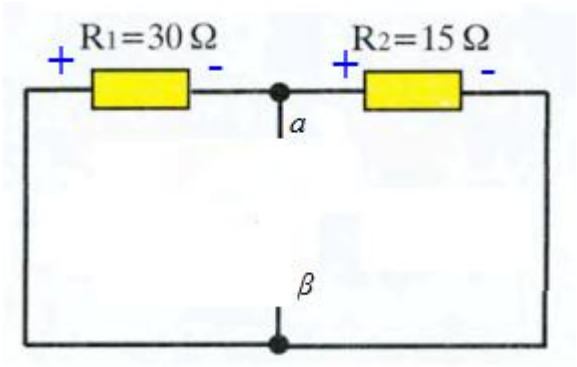
β' κανόνας του Kirchhoff στον βρόγχο:

$$-E_2 + R_2 I - U_{\alpha\beta} = 0 \Rightarrow$$

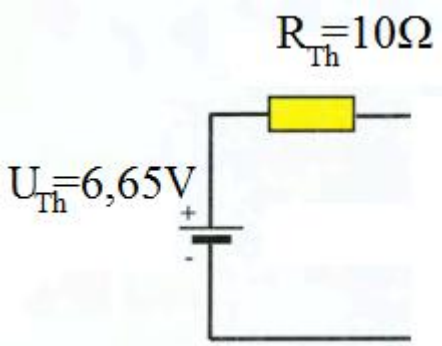
$$U_{\alpha\beta} = -E_2 + R_2 I = -60 + 15 * 3{,}555 = -6{,}66V$$

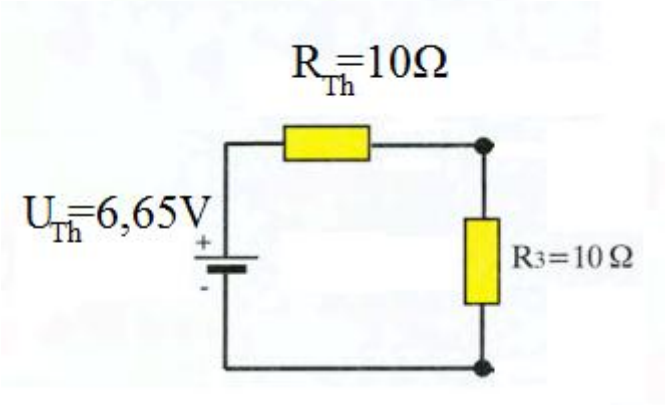
Υπολογιαμός της ισοδύναμης αντίστασης Thevenin





$$R_{\alpha\beta} = \frac{R_1 R_2}{R_1 + R_2} = \frac{30 * 15}{30 + 15} = 10\Omega$$





$$I = \frac{U_{Th}}{R_{Th} + R_3} = \frac{-6{,}65}{10 + 10} = -0{,}33A$$