ii) $\left(x+\frac{1}{x}\right)^2-2 = x^2+2\cdot x\cdot\frac{1}{x}+\left(\frac{1}{x}\right)^2-2 =$

$= x^2+2x\cdot\frac{1}{x}+\frac{1^2}{x^2}-2 =$

$= x^2+\frac{2x}{1}\cdot\frac{1}{x}+\frac{1}{x^2}-2 =$

$= x^2+\frac{2x}{x}+\frac{1}{x^2}-2 = x^2+2+\frac{1}{x^2}-2 = x^2+\frac{1}{x^2}$

iii) $\left(\frac{\alpha+\beta}{2}\right)^2+\left(\frac{\alpha-\beta}{2}\right)^2 =$

$= \frac{(\alpha+\beta)^2}{2^2}+\frac{(\alpha-\beta)^2}{2^2} = \frac{(\alpha+\beta)^2}{4}+\frac{(\alpha-\beta)^2}{4} =$

$= \frac{(\alpha+\beta)^2+(\alpha-\beta)^2}{4} = \frac{\alpha^2+2\alpha\beta+\beta^2+\alpha^2-2\alpha\beta+\beta^2}{4} =$

$= \frac{\alpha^2+\beta^2+\alpha^2+\beta^2}{4} = \frac{2\alpha^2+2\beta^2}{4} = \frac{2(\alpha^2+\beta^2)}{4} = \frac{\alpha^2+\beta^2}{2}$

iv) $\left(\frac{\alpha+\beta}{2}\right)^2-\left(\frac{\alpha-\beta}{2}\right)^2 = \frac{(\alpha+\beta)^2}{2^2}-\frac{(\alpha-\beta)^2}{2^2} =$

$= \frac{(\alpha+\beta)^2}{4}-\frac{(\alpha-\beta)^2}{4} = \frac{(\alpha+\beta)^2-(\alpha-\beta)^2}{4} =$

$= \frac{\alpha^2+2\alpha\beta+\beta^2-(\alpha^2-2\alpha\beta+\beta^2)}{4} = \frac{\alpha^2+2\alpha\beta+\beta^2-\alpha^2+2\alpha\beta-\beta^2}{4}$

$= \frac{2\alpha\beta+2\alpha\beta}{4} = \frac{4\alpha\beta}{4} = \alpha\beta.$