

$$\begin{aligned}
 \text{ii)} \quad \left(x + \frac{1}{x}\right)^2 - 2 &= x^2 + 2 \cdot x \cdot \frac{1}{x} + \left(\frac{1}{x}\right)^2 - 2 = \\
 &= x^2 + 2x \cdot \frac{1}{x} + \frac{1^2}{x^2} - 2 = \\
 &= x^2 + \frac{2x}{1} \cdot \frac{1}{x} + \frac{1}{x^2} - 2 = \\
 &= x^2 + \frac{2x}{x} + \frac{1}{x^2} - 2 = x^2 + 2 + \frac{1}{x^2} - 2 = x^2 + \frac{1}{x^2}
 \end{aligned}$$

$$\begin{aligned}
 \text{iii)} \quad \left(\frac{\alpha + \beta}{2}\right)^2 + \left(\frac{\alpha - \beta}{2}\right)^2 &= \\
 &= \frac{(\alpha + \beta)^2}{2^2} + \frac{(\alpha - \beta)^2}{2^2} = \frac{(\alpha + \beta)^2}{4} + \frac{(\alpha - \beta)^2}{4} = \\
 &= \frac{(\alpha + \beta)^2 + (\alpha - \beta)^2}{4} = \frac{\alpha^2 + 2\alpha\beta + \beta^2 + \alpha^2 - 2\alpha\beta + \beta^2}{4} = \\
 &= \frac{\alpha^2 + \beta^2 + \alpha^2 + \beta^2}{4} = \frac{2\alpha^2 + 2\beta^2}{4} = \frac{2(\alpha^2 + \beta^2)}{4} = \frac{\alpha^2 + \beta^2}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{iv)} \quad \left(\frac{\alpha + \beta}{2}\right)^2 - \left(\frac{\alpha - \beta}{2}\right)^2 &= \frac{(\alpha + \beta)^2}{2^2} - \frac{(\alpha - \beta)^2}{2^2} = \\
 &= \frac{(\alpha + \beta)^2}{4} - \frac{(\alpha - \beta)^2}{4} = \frac{(\alpha + \beta)^2 - (\alpha - \beta)^2}{4} = \\
 &= \frac{\alpha^2 + 2\alpha\beta + \beta^2 - (\alpha^2 - 2\alpha\beta + \beta^2)}{4} = \frac{\alpha^2 + 2\alpha\beta + \beta^2 - \alpha^2 + 2\alpha\beta - \beta^2}{4} \\
 &= \frac{2\alpha\beta + 2\alpha\beta}{4} = \frac{4\alpha\beta}{4} = \alpha\beta.
 \end{aligned}$$