

$x_0$

$\mu$   $\mu$  :.....

$\mu$   $\mu$  :.....

**A**

:

$$f(x) = \begin{cases} \frac{y-x^2}{x} \cdot \frac{1}{x}, x < 0 \\ \frac{|x^3 - x - 1| - 1}{x}, x > 0 \end{cases}$$

**A1.**  $\lim_{x \rightarrow 0} f(x)$  (μ 15+15)

**A2.**  $\lim_{x \rightarrow 0^+} [f(x) - 2f(-x)]$  (μ 20)

**B**

$f: \mathbb{R} \rightarrow \mathbb{R}$  :

$f^3(x) - x \cdot f^2(x) - x^2 \cdot f(-x) = x^2 \cdot \mu x, \quad x \in \mathbb{R}.$

$\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = r \in \mathbb{R}, \quad :$

**1.**  $f(0) = 0$  και  $\alpha = 1$  (μ 5+15)

**2.**  $\lim_{x \rightarrow 0} f(x) = 0$  (μ 10)

**3.** i)  $\lim_{x \rightarrow 0} \frac{f(f(x))}{x}$       ii)  $\lim_{x \rightarrow 1} \frac{f(x^2 - x)}{x^2 - 3x + 2}$   
(μ 5+15)

**A**

$$f(x) = \begin{cases} \frac{y-x^2}{x} \cdot \dagger \in \frac{1}{x}, x < 0 \\ \frac{|x^3-x-1|-1}{x}, x > 0 \end{cases}$$

**A1.**  $x < 0, f(x) = \frac{y-x^2}{x^2} \cdot x \cdot \dagger \in \frac{1}{x}, \dots \lim_{x \rightarrow 0^-} f(x) = 1 \cdot 0 = 0$

$x > 0, f(x) = \frac{|x^3-x-1|-1}{x} \quad x^3-x-1 < 0 \quad 0 \quad \mu ,$

$\lim_{x \rightarrow 0^+} (x^3-x-1) = -1 < 0$

$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} \frac{-x^3+x+1-1}{x} = \dots = 1$

$\lim_{x \rightarrow 0} f(x)$

**A2.**  $\lim_{x \rightarrow 0^+} [f(x) - 2f(-x)] = \lim_{x \rightarrow 0^+} f(x) - 2 \lim_{x \rightarrow 0^+} f(-x) = \dots = 1 - 2 = -1$

**B**

**1.**  $x = 0 \quad f^3(x) - x \cdot f^2(x) - x^2 \cdot f(-x) = x^2 \cdot \mu x, \quad f(0) = 0. \quad x = 0,$   
 $\mu \quad x^3 \quad 0 \quad \mu \quad f^3(x) - x \cdot f^2(x) - x^2 \cdot f(-x) = x^2 \cdot \mu x \dots$   
 $x \rightarrow 0, \quad \mu \quad \lim_{x \rightarrow 0} \frac{f(x)}{x} = a,$

$^3 - ^2 + -1 = 0 \Leftrightarrow \dots \Leftrightarrow = 1$

**2.** Θετόουμε  $\gamma \quad x \quad 0, \quad g(x) = \frac{f(x)}{x}, \quad f(x) = x \cdot g(x) \dots$

$\lim_{x \rightarrow 0} f(x) = 0$

**3.**  $\lim_{x \rightarrow 0} \frac{f(f(x))}{x} = \lim_{x \rightarrow 0} \frac{f(f(x))}{f(x)} \cdot \frac{f(x)}{x} = 1$

$\lim_{x \rightarrow 1} \frac{f(x^2-x)}{x^2-3x+2} = \lim_{x \rightarrow 1} \frac{f(x^2-x) \cdot (x^2-x)}{(x^2-x)(x^2-3x+2)} = \lim_{x \rightarrow 1} \frac{f(x^2-x)}{(x^2-x)} \cdot \lim_{x \rightarrow 1} \frac{(x^2-x)}{(x^2-3x+2)}$

$= 1 \cdot (-1) = -1$