Παράγωγος μερικών βασικών συναρτήσεων

Συνάρτηση	Παράγωγος
1. Αν $f(x)=c$	$f'(x)=0$
2. Αν $f(x)=x$	$f'(x)=1$
3. Αν $f(x)=x^{\nu}$	$f'(x)=\nu x^{\nu-1}$, $\nu \in R$
4. Αν $f(x)=\sqrt{x}$	$f'(x)=\frac{1}{2\sqrt{x}}$
5. Αν $f(x)=$ημx	$f'(x)=$συνx
6. Αν $f(x)=$συνx	$f'(x)=-$ημx
7. Αν $f(x)=e^{x}$	$f'(x)=e^{x}$
8. Αν $f(x)=\ln x$	$f'(x)=\frac{1}{x}$

$(x^{\nu})' = \nu \cdot x^{\nu-1}$

Παράγωγος μερικών σύνθετων συναρτήσεων

Απλή Συνάρτηση	Σύνθετη Συνάρτηση
1. Αφού $(x^\nu)' = \nu x^{\nu-1}$,	έχω $(u^\nu(x))' = \nu u^{\nu-1}(x)\,u'(x)$
2. Αφού $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$,	έχω $\left(\sqrt{u(x)}\right)' = \frac{1}{2\sqrt{u(x)}}u'(x)$
3. Αφού $(\eta\mu x)' = \sigma\upsilon\nu x$,	έχω $(\eta\mu u(x))' = \sigma\upsilon\nu u(x)u'(x)$
4. Αφού $(\sigma\upsilon\nu x)' = -\eta\mu x$,	έχω $(\sigma\upsilon\nu u(x))' = -\eta\mu u(x)u'(x)$
5. Αφού $(e^x)'=e^x$,	έχω $(e^{u(x)})'=e^{u(x)}\,u'(x)$
6. Αφού $(\alpha^x)'=\alpha^x\ln\alpha$,	έχω $(\alpha^{u(x)})'=\alpha^{u(x)}\ln\alpha\,u'(x)$
7. Αφού $(\ln x)'=\frac{1}{x}$,	έχω $(\ln u(x))'=\frac{1}{u(x)}\,u'(x)$

~~1)~~ $f(x) = (x^2 + 2x)^3$	~~2)~~ $f(x) = (\eta\mu x - \sigma\upsilon\nu x)^2$	~~3)~~ $f(x) = (x^2 - 3)^{2021}$
~~4)~~ $f(x) = \left(xe^x - \sqrt{2}\right)^2$	~~5)~~ $f(x) = (x - x^{-1})^2$	6) $f(x) = (3x - 1)^2$

$(u^3)' = 3u^2 \cdot u'$

$\underline{(u^2)' = 2u \cdot u'}$

$(u^{2021})' = 2021 \cdot u^{2020} \cdot u'$

$(fg)' = f'g + f \cdot g'$

$(x^{-1})' = -1 \cdot x^{-2} = -\frac{1}{x^2}$

1) $\left((x^2+2x)^3\right)' = 3(x^2+2x)^2 \cdot (x^2+2x)' = 3(x^2+2x)^2 \cdot (2x+2)$

$= 3 \cdot (x(x+2))^2 \cdot 2(x+1) = 3x^2 \cdot (x+2)^2 \cdot (x+1)$

2) $\left((\eta\mu x - \sigma\upsilon\nu x)^2\right)' = 2(\eta\mu x - \sigma\upsilon\nu x) \cdot (\eta\mu x - \sigma\upsilon\nu x)' = 2(\eta\mu x - \sigma\upsilon\nu x) \cdot (\sigma\upsilon\nu x + \eta\mu x)$

$= 2(\eta\mu^2 x - \sigma\upsilon\nu^2 x) = -2\sigma\upsilon\nu 2x$

3) $(x^2-3)^{2021} = 2021 \cdot (x^2-3)^{2020} \cdot (x^2-3)' = 2021 \cdot (x^2-3)^{2020} \cdot 2x$

$= 4.042 \cdot (x^2-3)^{2020} \cdot x$

4) $\left((xe^x - \sqrt{2})^2\right)' = 2(xe^x - \sqrt{2}) \cdot (xe^x - \sqrt{2})' = 2(xe^x - \sqrt{2}) \cdot (x'e^x + x(e^x)' - \cancel{\sqrt{2}'}^{\,0})$

$= 2(xe^x - \sqrt{2})(e^x + xe^x)$

$= 2e^x(xe^x - \sqrt{2})(1+x)$

5) $\left((x - x^{-1})^2\right)' = 2(x - x^{-1}) \cdot (x - x^{-1})'$

$= 2\left(x - \frac{1}{x}\right) \cdot \left(1 + \frac{1}{x^2}\right) = 2\left(\frac{x^2-1}{x} \cdot \frac{x^2+1}{x^2}\right) = 2 \cdot \frac{(x-1)(x+1)(x^2+1)}{x^3}$

6) $\left((3x-1)^2\right)' = 2(3x-1) \cdot (\cancel{3x-1)'}^{\,3} = 6 \cdot (3x-1).$

7) $f(x) = n\mu^2 x$	8) $f(x) = n\mu x^2$	9) $f(x) = n\mu 2x$
10) $f(x) = n\mu^2 x^2$	11) $f(x) = 3\eta\mu^5 2x$	12) $f(x) = \sigma\upsilon\nu^2 2x$

$(u^2)' = 2u \cdot u'$

$(\eta\mu u)' = \sigma\upsilon\nu u \cdot u'$

$(u^5)' = 5u^4 \cdot u'$

7) $(\eta\mu^2 x)' = 2 \cdot \eta\mu x \cdot (\eta\mu x)' = 2\eta\mu x \cdot \sigma\upsilon\nu x = \eta\mu 2x.$

8) $(\eta\mu x^2)' = \sigma\upsilon\nu x^2 \cdot (x^2)' = \underline{2x \cdot \sigma\upsilon\nu x^2}$

9) $(\eta\mu 2x)' = \sigma\upsilon\nu 2x \cdot (2x)' = 2\,\sigma\upsilon\nu 2x$

10) $(\eta\mu^2 x^2)' = 2\,\eta\mu x^2 \cdot (\eta\mu x^2)' = 2\,\eta\mu x^2 \cdot 2x \cdot \sigma\upsilon\nu x^2 = 2x \cdot \eta\mu 2x^2$

11) $3\eta\mu^5 2x = 3 \cdot 5 \cdot \eta\mu^4 2x \cdot (\eta\mu 2x)' = 15\,\eta\mu^4 2x \cdot \sigma\upsilon\nu 2x \cdot \cancel{(2x)'}_{2} = 30\,\eta\mu^4 2x \cdot \sigma\upsilon\nu 2x$

12) $(\sigma\upsilon\nu^2 2x)' = 2\,\sigma\upsilon\nu 2x \cdot (\sigma\upsilon\nu 2x)' = 2 \cdot \sigma\upsilon\nu 2x \cdot (-\eta\mu 2x) \cdot \cancel{(2x)'}_{2}$

$= -2 \cdot \underbrace{2\,\sigma\upsilon\nu 2x \cdot \eta\mu 2x}_{\eta\mu 4x}$

$= -2\,\eta\mu 4x$

~~13)~~ $f(x) = \eta\mu(\sigma\upsilon\nu x)$	~~14)~~ $f(x) = \eta\mu^2(\sigma\upsilon\nu x)$	~~15)~~ $f(x) = \eta\mu(x^2 - 2x)$
~~16)~~ $f(x) = \eta\mu\sqrt{x}$	~~17)~~ $f(x) = \eta\mu^3 2x$	18) $f(x) = \eta\mu(x^2+1)^2$

$(u^2)' = 2u \cdot u'$

$(\eta\mu u)' = \sigma\upsilon\nu u \cdot u'$

13) $\left(\eta\mu(\sigma\upsilon\nu x)\right)' = \sigma\upsilon\nu(\sigma\upsilon\nu x)\cdot(\sigma\upsilon\nu x)' = \sigma\upsilon\nu(\sigma\upsilon\nu x)\cdot(-\eta\mu x)$
$= -\eta\mu x \cdot \sigma\upsilon\nu(\sigma\upsilon\nu x)$

14) $\eta\mu^2(\sigma\upsilon\nu x) = 2\eta\mu(\sigma\upsilon\nu x)\cdot(\eta\mu(\sigma\upsilon\nu x))'$
$= 2\eta\mu(\sigma\upsilon\nu x)\cdot(-\eta\mu x)\cdot\sigma\upsilon\nu(\sigma\upsilon\nu x)$
$= -\eta\mu x \cdot \eta\mu(2\sigma\upsilon\nu x)$

15) $\left(\eta\mu(x^2-2x)\right)' = \sigma\upsilon\nu(x^2-2x)\cdot(x^2-2x)' = \sigma\upsilon\nu(x^2-2x)\cdot(2x-2)$
$= 2\cdot(x-1)\cdot\sigma\upsilon\nu^2(x^2-2x).$

16) $(\eta\mu\sqrt{x})' = \sigma\upsilon\nu\sqrt{x}\cdot(\sqrt{x})' = \dfrac{\sigma\upsilon\nu\sqrt{x}}{2\sqrt{x}}$

17) $(\eta\mu^3 2x)' = 3\eta\mu^2 2x\cdot(\eta\mu 2x)' = 3\eta\mu^2 2x\cdot\sigma\upsilon\nu 2x\cdot\cancel{(2x)'}2 = 6\eta\mu^2 2x\cdot\sigma\upsilon\nu 2x$

18) $\left(\eta\mu(x^2+1)^2\right)' = \sigma\upsilon\nu(x^2+1)^2\cdot\left((x^2+1)^2\right)' = \sigma\upsilon\nu(x^2+1)^2\cdot 2(x^2+1)\cdot(x^2+1)'$
$= \sigma\upsilon\nu(x^2+1)^2\cdot 2(x^2+1)\cdot 2x$

19) $f(x) = \eta\mu^2(x^2+1)$	**20)** $f(x) = \sigma\upsilon\nu(x^2+1)^2$	**21)** $f(x) = \sigma\upsilon\nu^3 x$
22) $f(x) = \sigma\upsilon\nu^2 x$	**23)** $f(x) = \sigma\upsilon\nu\, x^2$	**24)** $f(x) = \sigma\upsilon\nu 2x$

$\underline{(u^2)' = 2u\cdot u'}$

$\underline{(\sigma\upsilon\nu u)' = -\eta\mu u\cdot u'}$

$(u^3)' = 3u^2\cdot u'$

19) $(\eta\mu^2(x^2+1))' = 2\eta\mu(x^2+1)\cdot(\eta\mu(x^2+1))' = 2\eta\mu(x^2+1)\cdot\sigma\upsilon\nu(x^2+1)\cdot\underbrace{(x^2+1)'}_{2x}$

$= 4\cdot x\cdot\eta\mu(x^2+1)\,\sigma\upsilon\nu(x^2+1)$

20) $(\sigma\upsilon\nu(x^2+1)^2)' = -\eta\mu(x^2+1)^2\cdot((x^2+1)^2)' = -\eta\mu(x^2+1)^2\cdot 2(x^2+1)\cdot(x^2+1)'$

$= -\eta\mu(x^2+1)^2\cdot 2(x^2+1)\cdot 2x$

21) $(\sigma\upsilon\nu^3 x)' = 3\sigma\upsilon\nu^2 x(\sigma\upsilon\nu x)' = -3\sigma\upsilon\nu^2 x\cdot\eta\mu x$

22) $(\sigma\upsilon\nu^2 x)' = 2\sigma\upsilon\nu x(-\eta\mu x) = -\eta\mu 2x$

23) $(\sigma\upsilon\nu x^2) = -\eta\mu x^2\cdot(x^2)' = -2x\cdot\eta\mu x^2$

24) $(\sigma\upsilon\nu 2x)' = -\eta\mu 2x\cdot(2x)' = -2\eta\mu 2x$

25) $f(x) = \sigma\upsilon\nu^3\,(x^2)$	26) $f(x) = \sigma\upsilon\nu^2\,2x$	27) $f(x) = \sigma\upsilon\nu^2(\eta\mu x)$
28) $f(x) = \sqrt{x^2+2x}$	29) $f(x) = \sqrt{\eta\mu x}$	30) $f(x) = \sqrt{xe^x - ln2}$

$(u^3)' = 3u^2 \cdot u'$

$(\sigma\upsilon\nu u)' = -\eta\mu u \cdot u'$

$(u^2)' = 2u \cdot u'$

$(\sqrt{u})' = \frac{1}{2u} \cdot u'$

25) $\left(\sigma\upsilon\nu^3(x^2)\right)' = 3\,\sigma\upsilon\nu^2(x^2)\cdot\left(\sigma\upsilon\nu(x^2)\right)' = 3\,\sigma\upsilon\nu(x^2)\,(-\eta\mu\,x^2)\,(x^2)'$
$= -6x\,\sigma\upsilon\nu\,x^2\,\eta\mu\,x^2$

26) $(\sigma\upsilon\nu^2 x)' = 2\,\sigma\upsilon\nu x\,(\sigma\upsilon\nu x)' = 2\,\sigma\upsilon\nu x\,(-\eta\mu x) = -\eta\mu 2x$

27) $\left(\sigma\upsilon\nu^2(\eta\mu x)\right)' = 2\,\sigma\upsilon\nu(\eta\mu x)\cdot\left(\sigma\upsilon\nu(\eta\mu x)\right)' = \underbrace{2\,\sigma\upsilon\nu(\eta\mu x)\cdot(-\eta\mu(\eta\mu x))}\cdot(\eta\mu x)'$
$= -\eta\mu(2\eta\mu x)\cdot\sigma\upsilon\nu x$

28) $\left(\sqrt{x^2+2x}\right)' = \frac{(x^2+2x)'}{2\sqrt{x^2+2x}} = \frac{\cancel{2}x+\cancel{2}^{1}}{\cancel{2}\sqrt{x^2+2x}} = \frac{x+1}{\sqrt{x^2+2x}}$

29) $\left(\sqrt{\eta\mu x}\right)' = \frac{(\eta\mu x)'}{2\sqrt{\eta\mu x}} = \frac{\sigma\upsilon\nu x}{2\sqrt{\eta\mu x}}$

30) $\left(\sqrt{xe^x - \ell n2}\right)' = \frac{(xe^x - \ell n2)'}{2\sqrt{xe^x - \ell n2}} = \frac{\cancel{x'}^{1}e^x + x(e^x)' - \cancel{\ell n2'}^{0}}{2\sqrt{xe^x - \ell n2}} = \frac{e^x(1+x)}{2\sqrt{xe^x - \ell n2}}$

~~31)~~ $f(x)=\sqrt{\frac{x-1}{x+2}}$	~~32)~~ $f(x)=\sqrt{\sqrt{x}}$	~~33)~~ $f(x)=e^{\frac{1}{x}}$
~~34)~~ $f(x)=e^{-x}$	~~35)~~ $f(x)=\sqrt{ln\ x}$	~~36)~~ $f(x)=e^{\sqrt{x}}$

$(\sqrt{u})'=\frac{u'}{2\sqrt{u}}$

$\left(\frac{f}{g}\right)'=\frac{f'g-f\cdot g'}{g^2}$

$(x^v)'=v\cdot x^{v-1}$

$(e^u)'=e^u\cdot u'$

$(\ln x)'=\frac{1}{x}$

31) $\left(\sqrt{\frac{x-1}{x+2}}\right)'=\frac{\left(\frac{x-1}{x+2}\right)'}{2\sqrt{\frac{x-1}{x+2}}}=\frac{1}{2}\cdot\sqrt{\frac{x+2}{x-1}}\cdot\frac{\cancel{(x-1)'}(x+2)-(x-1)\cancel{(x+2)'}}{(x+2)^2}=\frac{1}{2}\sqrt{\frac{x+2}{x-1}}\cdot\frac{3}{(x+2)^2}$

32) $\left(\sqrt{\sqrt{x}}\right)'=\left((x^{1/2})^{1/2}\right)'=\left(x^{1/4}\right)'=\frac{1}{4}\cdot x^{\frac{1}{4}-1}=x^{-\frac{3}{4}}=\frac{1}{4}\cdot\frac{1}{\sqrt[4]{x^3}}$.

33) $\left(e^{\frac{1}{x}}\right)'=e^{\frac{1}{x}}\cdot\left(\frac{1}{x}\right)'=e^{\frac{1}{x}}\cdot\frac{-1}{x^2}=\frac{-e^{1/x}}{x^2}$

34) $(e^{-x})'=e^{-x}\cdot(-x)'=-e^{-x}$

35) $(\sqrt{\ln x})'=\frac{(\ln x)'}{2\sqrt{\ln x}}=\frac{1}{2x\sqrt{\ln x}}$

36) $(e^{\sqrt{x}})'=e^{\sqrt{x}}\cdot(\sqrt{x})'=\frac{e^{\sqrt{x}}}{2\sqrt{x}}$.

~~37)~~ $f(x) = e^{lnx}$	~~38)~~ $f(x) = e^{e^x}$	~~39)~~ $f(x) = e^{\eta\mu x - \sigma\upsilon\nu x}$
~~40)~~ $f(x) = e^{x\sqrt{x}}$	~~41)~~ $f(x) = e^{\sqrt{2}x}$	~~42)~~ $f(x) = e^{\eta\mu 2x}$

$(e^u)' = e^u \cdot u'$

$x\sqrt{x} = x^1 \cdot x^{1/2} = x^{3/2}$

37) $\left(e^{\ln x}\right)' = x' = 1$

38) $\left(e^{e^x}\right)' = e^{e^x} \cdot (e^x)' = e^{e^x} \cdot e^x = e^{e^x + x}$

39) $\left(e^{\eta\mu x - \sigma\upsilon\nu x}\right)' = e^{\eta\mu x - \sigma\upsilon\nu x} \cdot (\eta\mu x - \sigma\upsilon\nu x)' = e^{\eta\mu x - \sigma\upsilon\nu x} \cdot (\sigma\upsilon\nu x + \eta\mu x)$

40) $\left(e^{x\sqrt{x}}\right)' = \left(e^{x^{3/2}}\right)' = e^{x^{3/2}} \cdot (x^{3/2})' = e^{x\sqrt{x}} \cdot \frac{2}{3} \cdot x^{\frac{2}{3}-1}$

$= e^{x\sqrt{x}} \cdot \frac{2}{3} x^{-1/3} = \frac{2}{3} e^{x\sqrt{x}} \cdot \frac{1}{\sqrt[3]{x}}$

41) $\left(e^{\sqrt{2}\cdot x}\right)' = e^{\sqrt{2}\cdot x} (\sqrt{2}x)' = \sqrt{2}\, e^{\sqrt{2}x}$

42) $\left(e^{\eta\mu 2x}\right)' = e^{\eta\mu 2x} \cdot (\eta\mu 2x)' = e^{\eta\mu 2x} \cdot \sigma\upsilon\nu 2x \; (2x)'$

$= 2\, e^{\eta\mu 2x} \cdot \sigma\upsilon\nu 2x$

~~43)~~ $f(x) = e^{-x^2}$	~~44)~~ $f(x) = e^{xe^x}$	~~45)~~ $f(x) = e^{x\ln x}$
~~46)~~ $f(x) = e^{x-\ln x}$	~~47)~~ $f(x) = \ln\,(n\mu x)$	~~48)~~ $f(x) = \ln(n\mu^2 x)$

$(\ln u)' = \frac{u'}{u}$

$(u^2)' = 2u\cdot u'$

43) $\left(e^{-x^2}\right)' = e^{-x^2}\left(-x^2\right)' = -2x\,e^{-x^2}$

44) $\left(e^{x\cdot e^x}\right)' = e^{xe^x}\cdot(xe^x)' = e^{xe^x}\cdot\left(x'e^x + x(e^x)'\right) = e^{xe^x}\cdot\left(e^x + x\cdot e^x\right)$
$= e^{xe^x}\cdot e^x\,(1+x) = e^{xe^x+x}\cdot(1+x)$

45) $\left(e^{x\ln x}\right)' = e^{x\ln x}(x\ln x)' = e^{x\ln x}\cdot\left(\overset{1}{\cancel{x'}}\ln x + \cancel{x}\,\overset{\frac{1}{\cancel{x}}}{\cancel{(\ln x)'}}\right) = e^{x\ln x}(\ln x + 1)$

46) $\left(e^{x-\ln x}\right)' = e^{x-\ln x}(x-\ln x)' = e^{x-\ln x}\left(1-\frac{1}{x}\right)$

47) $\ln(\eta\mu x) = \frac{(\eta\mu x)'}{\eta\mu x} = \frac{\sigma\upsilon\nu x}{\eta\mu x} = \sigma\varphi x$

48) $\ln(\eta\mu^2 x) = \frac{(\eta\mu^2 x)'}{\eta\mu^2 x} = \frac{2\cancel{\eta\mu x}\cdot(\eta\mu x)'}{\cancel{\eta\mu x}\cdot\eta\mu x} = \frac{2\,\sigma\upsilon\nu x}{\eta\mu x} = 2\,\sigma\varphi x$

49) $f(x) = ln\sqrt{x}$	50) $f(x) = ln\,(n\mu 2x)$	51) $f(x) = ln(\sigma\upsilon v 2x - 1)$
52) $f(x) = ln(n\mu x - \sigma\upsilon v x)$	53) $f(x) = ln\left(x\sqrt{x}\right)$	54) $f(x) = x ln e^{x}$

$(\ln u)' = \frac{u'}{u}$

49) $(\ln\sqrt{x})' = (\ln x^{1/2})' = \left(\frac{1}{2}\ln x\right)' = \frac{1}{2x}$

50) $(\ln(\eta\mu 2x))' = \frac{(\eta\mu 2x)'}{\eta\mu 2x} = \frac{\sigma\upsilon\nu 2x\,(2x)'}{\eta\mu 2x} = 2\cdot\sigma\varphi 2x$

51) $(\ln(\sigma\upsilon\nu 2x - 1))' = \frac{(\sigma\upsilon\nu 2x - 1)'}{\sigma\upsilon\nu 2x - 1} = \frac{-\eta\mu 2x\cdot(2x)'}{\cancel{1} - 2\eta\mu^2 x - \cancel{1}} = \frac{\cancel{-2}\cdot\cancel{2}\eta\mu x\cdot\sigma\upsilon\nu x}{\cancel{-2}\,\eta\mu x\cdot\eta\mu x} = 2\sigma\varphi x$

52) $(\ln(\eta\mu x - \sigma\upsilon\nu x))' = \frac{(\eta\mu x - \sigma\upsilon\nu x)'}{\eta\mu x - \sigma\upsilon\nu x} = \frac{\sigma\upsilon\nu x + \eta\mu x}{\eta\mu x - \sigma\upsilon\nu x}$

53) $(\ln(x\sqrt{x}))' = (\ln x^{3/2})' = \left(\frac{3}{2}\ln x\right)' = \frac{3}{2x}$

54) $(x\cdot\ln e^{x})' = (x\cdot x)' = (x^2)' = 2x$

55) $f(x) = ln(xe^x)$	56) $f(x) = ln\,(e^x - x)$	57) $f(x) = ln(lnx)$
58) $f(x) = ln(2x^2 - 3)$	59) $f(x) = ln\,(x^2 + 1)$	60) $f(x) = ln\left(x + \frac{1}{x}\right)$

55) $\left(ln(xe^x)\right)' = \frac{(xe^x)'}{xe^x} = \frac{(x+1)\cdot \cancel{e^x}}{x\,\cancel{e^x}} = 1 + \frac{1}{x}$

56) $\left(ln(e^x - x)\right)' = \frac{(e^x - x)'}{e^x - x} = \frac{e^x - 1}{e^x - x}$

57) $\left(ln(lnx)\right)' = \frac{(lnx)'}{ln(lnx)} = \frac{1}{x\,ln(lnx)}$

58) $\left(ln(2x^2-3)\right)' = \frac{(2x^2-3)'}{2x^2-3} = \frac{4x}{2x^2-3}$

59) $\left(ln(x^2+1)\right)' = \frac{(x^2+1)'}{x^2+1} = \frac{2x}{x^2+1}$

60) $\left(ln\left(x+\frac{1}{x}\right)\right)' = \frac{\left(x+\frac{1}{x}\right)'}{x+\frac{1}{x}} = \frac{1-\frac{1}{x^2}}{x+\frac{1}{x}} = \frac{x^2-1}{x^3+x} = \frac{(x-1)(x+1)}{x(x^2+1)}$

~~61)~~ $f(x) = ln(x\,lnx)$	~~62)~~ $f(x) = ln\,(ln\,(ln\;x))$	~~63)~~ $f(x) = e^{-lnx}$
~~64)~~ $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$	~~65)~~ $f(x) = e^{\frac{x-1}{x-2}}$	66) $f(x) = \sqrt{\frac{x-2}{x+3}}$

61) $\left(\ln(x\ln x)\right)' = \frac{(x\ln x)'}{x\ln x} = \frac{\ln x + 1}{x.\ln x}$

62) $\left(\ln(\ln(\ln x))\right)' = \frac{(\ln(\ln x))'}{\ln(\ln(\ln x))} = \frac{\frac{(\ln x)'}{\ln(\ln x)}}{\ln(\ln(\ln x))} = \frac{\frac{1}{x\ln(\ln x)}}{\ln(\ln(\ln x))}$

$= \frac{1}{x.\ln(\ln x).\ln(\ln(\ln(x)))}$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

63) $\left(e^{-\ln x}\right)' = \left(e^{\ln x^{-1}}\right)' = (x^{-1})' = -\frac{1}{x^2}$

64) $\left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)' = \frac{(e^x - e^{-x})'.(e^x + e^{-x}) - (e^x - e^{-x})(e^x + e^{-x})'}{(e^x + e^{-x})^2}$

$= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2} = \frac{\cancel{e^{2x}} + 2e^xe^{-x} + \cancel{e^{-2x}} - (\cancel{e^{2x}} - 2e^xe^{-x} + \cancel{e^{-2x}})}{(e^x + e^{-x})^2}$

$(e^u)' = e^u.u'$

$= \frac{4}{(e^x + e^{-x})^2}$

65) $\left(e^{\frac{x-1}{x-2}}\right)' = e^{\frac{x-1}{x-2}}.\left(\frac{x-1}{x-2}\right)' = e^{\frac{x-1}{x-2}}.\frac{(x-1)'(x-2) - (x-1)(x-2)'}{(x-2)^2} = \frac{-e^{\frac{x-1}{x-2}}}{(x-2)^2}$

$(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$

66) $\left(\sqrt{\frac{x-2}{x+3}}\right)' = \frac{\left(\frac{x-2}{x+3}\right)'}{2\sqrt{\frac{x-2}{x+3}}} = \frac{1}{2}.\sqrt{\frac{x+3}{x-2}}.\frac{(x-2)'(x+3) - (x-2)(x+3)'}{(x+3)^2}$

$= \frac{5}{2}\sqrt{\frac{x+3}{x-2}}.\frac{1}{(x+3)^2}$

67) $f(x) = (x+1)\cdot ln(x+1) - x$	**68)** $f(x) = e^{\sqrt{x^2+3}}$	**69)** $f(x) = ln(\sigma\upsilon\nu^2 x + e^{2x})$
70) $f(x) = \eta\mu^2(e^{x-2})$	**71)** $f(x) = 5^{\eta\mu x}$	**72)** $f(x) = \frac{ln(2x+1)}{ln(3x+1)}$

67) $\left((x+1)(\ln(x+1)) - x\right)' = (x+1)'\cdot\ln(x+1) + \cancel{(x+1)}\,\frac{1}{\cancel{x+1}} - 1$
$= \ln(x+1)$

68) $\left(e^{\sqrt{x^2+3}}\right)' = e^{\sqrt{x^2+3}}\left(\sqrt{x^2+3}\right)' = e^{\sqrt{x^2+3}}\cdot\frac{(x^2+3)'}{2\sqrt{x^2+3}} = \frac{e^{\sqrt{x^2+3}}\cdot\cancel{2}x}{\cancel{2}\sqrt{x^2+3}}$

$(\ln u)' = \frac{u'}{u}$

$(u^2)' = 2u\cdot u'$

69) $\left(\ln(\sigma\upsilon\nu^2 x + e^{2x})\right)' = \frac{(\sigma\upsilon\nu^2 x + e^{2x})'}{\sigma\upsilon\nu^2 x + e^{2x}} = \frac{2\sigma\upsilon\nu x(\sigma\upsilon\nu x)' + e^{2x}(2x)'}{\sigma\upsilon\nu^2 x + e^{2x}}$

$= \frac{-2\eta\mu x\,\sigma\upsilon\nu x + 2e^{2x}}{\sigma\upsilon\nu^2 x + e^{2x}}$

$(a^x)' = a^x\cdot\ln a$

70) $\left(\eta\mu^2(e^{x-2})\right)' = 2\eta\mu(e^{x-2})\cdot\left(\eta\mu(e^{x-2})\right)' = \underbrace{2\eta\mu(e^{x-2})\cdot\sigma\upsilon\nu(e^{x-2})}\cdot(e^{x-2})'$
$= \eta\mu(2e^{x-2})\cdot e^{x-2}$

71) $\left(5^{\eta\mu x}\right)' = 5^{\eta\mu x}\ln 5\,(\eta\mu x)' = 5^{\eta\mu x}\cdot\ln 5\cdot\sigma\upsilon\nu x$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

72) $\left(\frac{\ln(2x+1)}{\ln(3x+1)}\right)' = \frac{(\ln(2x+1))'\ln(3x+1) - \ln(2x+1)\cdot(\ln(3x+1))'}{\ln^2(3x+1)}$

$= \frac{\frac{(2x+1)'}{2x+1}\cdot\ln(3x+1) - \ln(2x+1)\cdot\frac{(3x+1)'}{3x+1}}{\ln^2(3x+1)}$

$= \frac{\frac{2\ln(3x+1)}{2x+1} - \frac{3\ln(2x+1)}{3x+1}}{\ln^2(3x+1)}$

73) $f(x) = ln\|x^2-2\|$	74) $f(x) = xe^{\sqrt{-x}}$	75) $\omega(x) = 2^x x^2$
76) $f(x) = e^{x^2+2}(x^4-4x^2)$	77) $f(t) = e^{t^t-t}$	78) $s(t) = \frac{1}{2}t^2+5t-3$

$(\ln|u|)' = \frac{u'}{u}$

$(fg)' = f'g + fg'$

$(e^u)' = e^u \cdot u'$

$(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$

$\frac{x}{\sqrt{-x}} = \frac{(\sqrt{-x})^2}{\sqrt{-x}} = \sqrt{-x}$

$t^t = e^{\ln t^t} = e^{t\ln t}$

73) $(\ln|x^2-2|)' = \frac{(x^2-2)'}{x^2-2} = \frac{2x}{x^2-2}$

74) $(xe^{\sqrt{-x}})' = x'e^{\sqrt{-x}} + x(e^{\sqrt{-x}})' = e^{\sqrt{-x}} + xe^{\sqrt{-x}}\cdot(\sqrt{-x})'$

$= e^{\sqrt{-x}} + x\frac{e^{\sqrt{-x}}\cdot(-x)'}{2\sqrt{-x}} = e^{\sqrt{-x}}\left(1 - \frac{x}{2\sqrt{-x}}\right) = e^{\sqrt{-x}}(1-\sqrt{-x})$

75) $(2^x x^2)' = (2^x)'x^2 + 2^x(x^2)' = 2^x\ln 2\cdot x^2 + 2^x\cdot 2x = 2^x\cdot x(x\ln 2+2)$

76) $(e^{x^2+2}(x^4-4x^2))' = (e^{x^2+2})'(x^4-4x^2) + e^{x^2+2}\cdot(x^4-4x^2)'$

$= e^{x^2+2}\cdot\underbrace{(x^2+2)'}_{2x}(x^4-4x^2) + e^{x^2+2}(4x^3-8x)$

$= e^{x^2+2}(2x^5-8x^3) + e^{x^2+2}(4x^3-8x) = e^{x^2+2}(2x^5-4x^3-8x)$

77) $(e^{t^t-t})' = e^{t^t-t}(t^t-t)' = e^{t^t-t}\cdot(e^{t\ln t}-t)'$

$= e^{t^t-t}\cdot(e^{t\ln t}(t\ln t)' - 1) = e^{t^t-t}(t^t(\ln t+1)-1)$

78) $(\frac{1}{2}t^2+5t-3)' = \frac{1}{2}\cdot 2t+5 = t+5$

79) $P(t) = t \cdot e^{-2t+1}$	**80)** $V(t) = \frac{a}{\ln t}$	**81)** $E(r) = \pi(r+1)(r+2)$
82) $f(R) = \frac{R \cdot R_1}{R+R_1}$	**83)** $s(t) = u_0 t + \frac{1}{2} a t^2$	**84)** $u(t) = u_0 + at$

$\left(\frac{1}{u}\right)' = \frac{-u'}{u^2}$

79) $\left(t e^{-2t+1}\right)' = t' e^{-2t+1} + t\left(e^{-2t+1}\right)' = e^{-2t+1} + t \cdot e^{-2t+1} \cancel{(-2t+1)'} \to -2$

$= e^{-2t+1}(1-2t)$

80) $\left(\frac{a}{\ln t}\right)' = -\frac{a(\ln t)'}{\ln^2 t} = \frac{-a}{t \ln^2 t}$

$(fg)' = f'g + fg'$

81) $\left(\pi(r+1)(r+2)\right)' = \pi \cancel{(r+1)'} \to 1 \, (r+2) + \pi (r+1) \cancel{(r+2)'} \to 1$

$= \pi(2r+3)$

82) $\left(\frac{R \cdot R_1}{R+R_1}\right)' = \frac{(RR_1)'(R+R_1) - RR_1 \cancel{(R+R_1)'} \to 1}{(R+R_1)^2} = \frac{R_1(\cancel{R}+R_1) - \cancel{RR_1}}{(R+R_1)^2} = \frac{R_1^2}{(R+R_1)^2}$

83) $\left(u_0 t + \frac{1}{2} a t^2\right)' = u_0 + \frac{1}{\cancel{2}} a \cancel{2} t = u_0 + at$

84) $(u_0 + at)' = \cancel{u_0'} \to 0 + (at)' = a \cdot 1 = a$

ΤΕΛΟΣ

ΚΑΛΗ

ΧΡΟΝΙΑ