ΘΕΜΑ Α

ΑΠΑΝΤΗΣΕΙΣ

Α1: $\bar{X} = \frac{t_1 - \bar{x} + t_2 - \bar{x} + \dots + t_v - \bar{x}}{v} =$

$= \frac{\sum_{i=1}^{v} t_i - v \cdot \bar{x}}{v}$

$= \frac{\sum_{i=1}^{v} t_i}{v} - \bar{x} = \bar{x} - \bar{x} = 0.$

Α2: $\bar{x}_w = \frac{\sum_{i=1}^{v} x_i w_i}{\sum_{i=1}^{v} w_i}$.

Α3: Ω: βέβαιο ενδεχόμενο ο δ.χ.
$\emptyset$: αδύνατο -"-

Α4: α) Σ
β) Λ $\quad (\sqrt{x})' = \frac{1}{2\sqrt{x}}$
γ) Σ
δ) Λ
ε) Λ

ΘΕΜΑ Β

$$f(x) = 2\sqrt{x^2-x+1} - 1, \quad x \in \mathbb{R}$$

Β1. $\lim\limits_{x\to 1} \dfrac{f(x)-1}{x-1}$

$x^2 - x + 1 > 0 \quad \forall x \in \mathbb{R}$.

$\Delta = 1 - 4 = -3 < 0$

Για $x \neq 1$: $\lim\limits_{x\to 1} \dfrac{2\sqrt{x^2-x+1} - 1 - 1}{x-1} =$

$= \lim\limits_{x\to 1} \dfrac{2\sqrt{x^2-x+1} - 2}{x-1} = \lim\limits_{x\to 1} \dfrac{2\left(\sqrt{x^2-x+1} - 1\right)}{x-1} =$

$= \lim\limits_{x\to 1} \dfrac{2\cdot\left(\sqrt{x^2-x+1} - 1\right)\left(\sqrt{x^2-x+1} + 1\right)}{(x-1)\left(\sqrt{x^2-x+1} + 1\right)} =$

$= \lim\limits_{x\to 1} \dfrac{2\left(x^2-x+1\right) - 1}{(x-1)\left(\sqrt{x^2-x+1} + 1\right)} = \lim\limits_{x\to 1} \dfrac{2\,x\,(x-1)}{(x-1)\left(\sqrt{x^2-x+1} + 1\right)} =$

$= \dfrac{2\cdot 1}{\sqrt{1-1+1} + 1} = \dfrac{2}{2} = 1.$

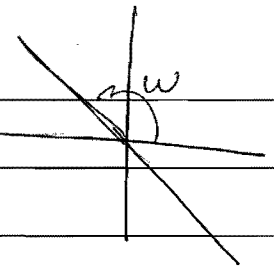
Β2. $f'(x) = \left(2\sqrt{x^2-x+1} - 1\right)' = 2\cdot\dfrac{1}{2\sqrt{x^2-x+1}}\left(x^2-x+1\right)' =$

$= \dfrac{2x-1}{\sqrt{x^2-x+1}}$

$f'(0) = \dfrac{-1}{1} = -1$

$\epsilon\varphi\omega = f'(0) = -1$

$\omega = \frac{3\pi}{4}$ ή $135°$



ΘΕΜΑ Γ (*) Βλ. Τέλος

$\frac{14}{30}$ | $\frac{25}{18}$ | 4

420 | 200 | $\frac{25}{2}$

Αρ. Βαθ.	Κέντρο x_i	Συχν. v_i	$x_i v_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$(x_i - \bar{x})^2 v_i$
[0 – 4)	2	20	40	−8	64	1280
[4 – 8)	6	10 40	240	−4	16	640
[8 – 12)	~~10~~	45	450	0	0	0
[12 – 16)	14	15 30	420	4	16	480
[16 – 20]	18	25	450	8	64	1600
Σύνολο		160	1600			4000

Γ_1: $c = \frac{R}{5} = \frac{20-0}{5} = 4$

Γ_2: $\bar{x} = \frac{1600}{160} = 10$.

$S^2 = \frac{4000}{160} = 25$

$S = 5$

4 5 6 7 8

$\frac{1}{4}40 = 10$

Γ_3: $CV = \frac{S}{|\bar{x}|} = \frac{5}{10} = \frac{1}{2} = 0{,}5$ ή $50\% > 10\%$

ομοιογενές

Γ4: $7 \le X \le 14$: $\nu = 10 + 45 + 15 = 70$

$$P(7 \le X \le 14) = \frac{44}{160} = \frac{7}{16} = 0,4375$$

43,75%

ΘΕΜΑ Δ

$$f(x) = \ln[x - P(A)] - \frac{1}{2}[x - P(A)]^2 + P(B), \quad x > P(A)$$

$0 \le P(A) \le 1$

Δ1:

$$f'(x) = \frac{1}{x - P(A)}(x - P(A))' - \frac{1}{2} \cdot 2(x - P(A)) \cdot (x - P(A))'$$

$$= \frac{1}{x - P(A)} - (x - P(A)) =$$

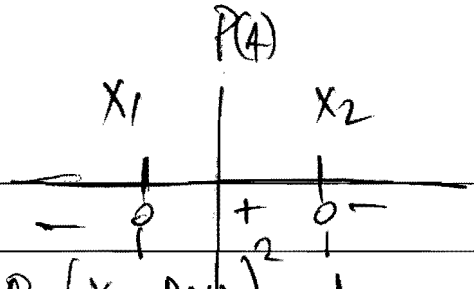
$$= \frac{1 - (x - P(A))^2}{x - P(A)}.$$

$$f'(x) \ge 0 \Leftrightarrow \frac{1 - (x - P(A))^2}{x - P(A)} \ge 0,$$

$x - P(A) > 0$ $\forall x$

$\Leftrightarrow 1 - (x - P(A))^2 \ge 0$

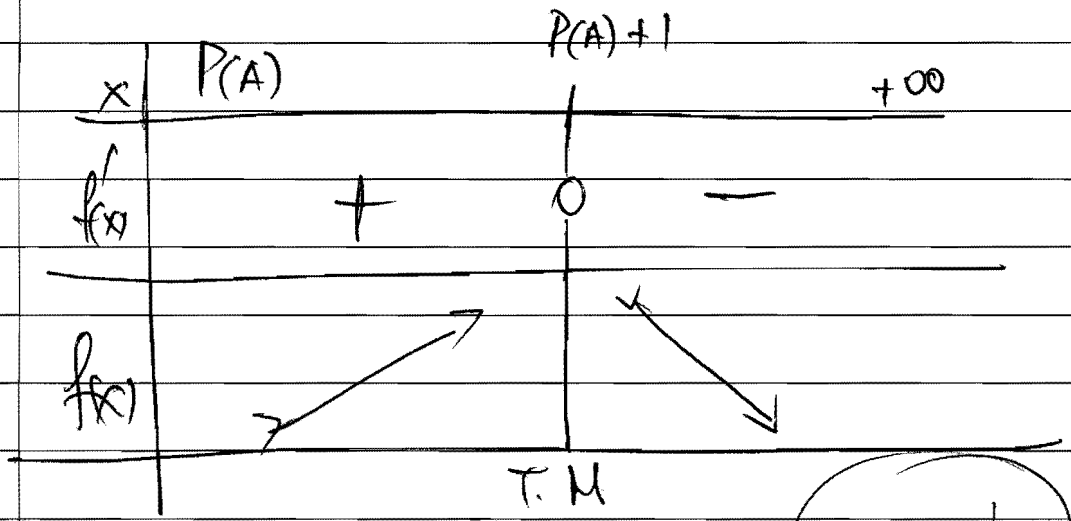
$(1 - x + P(A))(1 + x - P(A)) \ge 0$



$$\left(x - P(A)\right)^2 = 1$$

$$x - P(A) = 1 \quad \text{ou} \quad x - P(A) = -1$$

$$x_2 = P(A) + 1 \quad \text{ou} \quad x_1 = P(A) - 1$$



$$y = f(P(A)+1) = P(B) - \frac{1}{2}$$

$$= \ln\left[P(A)+1-P(A)\right] - \frac{1}{2}\left[P(A)+1-P(A)\right]^2 + P(B)$$

$$= \ln 1 - \frac{1}{2}\cdot 1 + P(B)$$

$$= -\frac{1}{2} + P(B)$$

△2. $x_0 = \frac{5}{3} = P(A)+1 \iff P(A) = \frac{5}{3} - \frac{3}{3} = \frac{2}{3}$

$$f\left(\frac{5}{3}\right) = 0 \iff P(B) - \frac{1}{2} = 0 \iff P(B) = \frac{1}{2}.$$

A3. $P(A \cup B) = \frac{5}{6}$

$P(A) = \frac{2}{3}$

$P(B) = \frac{1}{2}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P[(A \cap B)'] = ?$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{2}{3} + \frac{1}{2} - \frac{5}{6}$

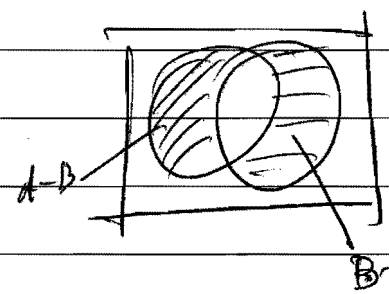
$= \frac{4 + 3 - 5}{6} = \frac{7-5}{6} = \frac{2}{6} =$

$P[(A \cap B)'] = 1 - P(A \cap B) =$

$= 1 - \frac{1}{3} = \frac{2}{3}$ ✓

$= \frac{1}{3}$ ✓

A4 $P[(A - B) \cup (B - A)]$



$(A - B) \cap (B - A) = \emptyset$

$P[(A - B) \cup (B - A)] = P(A - B) + P(B - A) =$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B) =$

$= P(A) + P(B) - 2P(A \cap B) =$

$= \frac{2}{3} + \frac{1}{2} - \frac{2 \cdot 1}{3} = \frac{1}{2}$

MTJ

∴ Από τον κύκλο C:

$$\frac{X_A + X_B}{2} = 6 \quad \Leftrightarrow \quad X_A + X_B = 12$$

$$\frac{0 + X_A}{2} = 6 - (X_B - X_A) \quad \Leftrightarrow \quad X_A = 12 - 2X_B + 2X_A$$

$$X_A + X_B = 12 - X_B + 2X_A$$

$$2X_A = X_B$$

$$\frac{X_A + 2X_A}{2} = 6 \quad \Leftrightarrow \quad X_A + 2X_A = 12$$

$$3X_A = 12$$

$$X_A = 4$$

$$X_B = 2 \cdot 4 = 8$$

$$C = X_B - X_A = 8 - 4 = 4$$