ΜΑΘΗΜΑΤΙΚΟ ΤΥΠΟΛΟΓΙΟ

Çëéáó Óêáñäáíáó - Ìáèçìáôéêïó .

Σταθερές.

π	=	03,14159 26535 89793 23846 2643...
e	=	02,71828 18284 59045 23536 0287...
e^{π}	=	23,14069 26327 79269 006...
π^{e}	=	22,45915 77183 61045 47342 715...
e^{e}	=	15,15426 22414 79264 190...
$\sqrt{2}$	=	01,41421 35623 73095 0488...
$\sqrt{3}$	=	01,73205 08075 68877 2935...
$\sqrt{5}$	=	02,23606 79774 99789 6964...
$\sqrt{e}$	=	01,64872 12707 00128 1468...
$\sqrt{\pi}$	=	01,77245 38609 05516 02729 8167...
log2	=	00,30102 99956 63981 19521 37389...
log3	=	00,47712 12547 19662 43729 50279...
loge	=	00,43429 44819 03251 82765...
logπ	=	00,49714 98726 94133 85435 12683...
ln2	=	00,69314 71805 59945 30941 7232...
ln3	=	01,09861 22886 68109 69139 5245...
ln10	=	02,30258 50929 94045 68401 7991...
lnπ	=	01,14472 9886...

Μαθηματική λογική.

P	q	$\overline{p}$	$p \wedge q$	$p \vee q$	$p \underline{\vee} q$	$p \Rightarrow q$	$p \Leftrightarrow q$
α	α	ψ	α	α	ψ	α	α
α	ψ	ψ	ψ	α	α	ψ	ψ
ψ	α	α	ψ	α	α	α	ψ
ψ	ψ	α	ψ	ψ	ψ	α	α

- $(p \Rightarrow q) \Leftrightarrow (\overline{q} \Rightarrow \overline{p})$

Σύνολα.

- $A \subseteq B \Leftrightarrow [x \in A \Rightarrow x \in B]$
- $A \subset B \Leftrightarrow [(x \in A \Rightarrow x \in B) \wedge \exists x \in B \therefore x \notin A]$
  $\Leftrightarrow [A \subseteq B \wedge \exists x \in B \therefore x \notin A]$

- $A=B \Leftrightarrow [(A \subseteq B) \wedge (B \subseteq A)]$
- $A \cap B \equiv \{x \therefore x \in A \wedge x \in B\}$.
- $A \cup B \equiv \{x : x \in A \vee x \in B\}$.
- $A-B \equiv \{x \in A \therefore x \notin B\}$.
- $A^c \equiv U-A=\{x \in U \therefore x \notin B\}$.
- $A \dot{+} B \equiv (A-B) \cup (B-A)$.
- $A \cap \varnothing = \varnothing$ και $A \cup \varnothing = A$, $\forall A$.
- $A \cap A = A$ και $A \cup A = A$, $\forall A$.
- $A \cap U = A$ και $A \cup U = U$, $\forall A$.
- $A \cap (B \cap \Gamma) = (A \cap B) \cap \Gamma$ και $A \cup (B \cup \Gamma) = (A \cup B) \cup \Gamma$ $\forall A,B,\Gamma$
- $A \cap B = B \cap A$ και $A \cup B = B \cup A$, $\forall A,B$
- $A \cup (B \cap \Gamma) = (A \cup B) \cap (A \cup \Gamma)$, $\forall A,B,\Gamma$
- $A \cap (B \cup \Gamma) = (A \cap B) \cup (A \cap \Gamma)$, $\forall A,B,\Gamma$

Συμβολισμοί και ιδιότητές τους.

- $\nu! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot \nu$ με $\nu \in \mathbb{N}^*$ και $0!=1$.
- $\binom{\nu}{\kappa} = \frac{\nu!}{\kappa!(\nu-\kappa)!}$, $\nu,\kappa \in \mathbb{N}$.
- $\binom{\nu}{\kappa} + \binom{\nu}{\kappa+1} = \binom{\nu+1}{\kappa+1}$, $\nu,\kappa \in \mathbb{N}$.
- $\sum_{i=1}^{\nu} x_i = x_1+x_2+x_3+\ldots+x_\nu$.
- $\sum_{i=1}^{\nu} \lambda x_i = \lambda \cdot \sum_{i=1}^{\nu} x_i$
- $\sum_{i=1}^{\nu} (x_i+y_i) = \sum_{i=1}^{\nu} x_i + \sum_{i=1}^{\nu} y_i$
- $\sum_{i=1}^{\nu} x = \nu \cdot x$.

Αξιοσημείωτες ταυτότητες.

- $(\alpha+\beta)^2=\alpha^2+2\alpha\beta+\beta^2$.
- $(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2$.
- $(\alpha+\beta)^3=\alpha^3+3\alpha^2\beta+3\alpha\beta^2+\beta^3=\alpha^3+\beta^3+3\alpha\beta(\alpha+\beta)$.
- $(\alpha-\beta)^3=\alpha^3-3\alpha^2\beta+3\alpha\beta^2-\beta^3=\alpha^3-\beta^3-3\alpha\beta(\alpha-\beta)$.
- $(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2\alpha\beta+2\beta\gamma+2\gamma\alpha$.
- $(\alpha+\beta+\gamma)^3=\alpha^3+\beta^3+\gamma^3+3(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$.
- $(\alpha+\beta)(\alpha-\beta)=\alpha^2-\beta^2$.
- $(x-\alpha)(x-\beta)=x^2-(\alpha+\beta)x+\alpha\beta$.
- $\alpha^\nu-\beta^\nu=(\alpha-\beta)\cdot(\alpha^{\nu-1}+\alpha^{\nu-2}\beta+\alpha^{\nu-3}\beta^2+\ldots+\alpha\beta^{\nu-2}+\beta^{\nu-1})$, $\forall\nu\in\mathbb{N}^*$.
- $\alpha^\nu+\beta^\nu=(\alpha+\beta)\cdot(\alpha^{\nu-1}-\alpha^{\nu-2}\beta+\alpha^{\nu-3}\beta^2-\ldots-\alpha\beta^{\nu-2}+\beta^{\nu-1})$, $\forall\nu\in\mathbb{N}^*\therefore\nu=2\kappa+1$
- $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma=(\alpha+\beta+\gamma)\cdot(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)$.
- $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma=\frac{1}{2}(\alpha+\beta+\gamma)\cdot[(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2]$.
- $(\alpha+\beta)^\nu=\sum_{\kappa=0}^{\nu}\binom{\nu}{\kappa}\cdot\alpha^{\nu-\kappa}\cdot\beta^\kappa$

Χρήσιμες ανισότητες.

- $x^2\geq 0$, $\forall x\in\mathbb{R}$.
- $\alpha^2+\beta^2\geq 2\alpha\beta$ και $\alpha^2+\beta^2\geq -2\alpha\beta$ $\forall\alpha,\beta\in\mathbb{R}$.
- $\alpha^2+\beta^2\geq\alpha\beta$ και $\alpha^2+\beta^2\geq-\alpha\beta$ $\forall\alpha,\beta\in\mathbb{R}$.
- $\alpha^2+\beta^2+\gamma^2\geq\alpha\beta+\beta\gamma+\gamma\alpha$. $\forall\alpha,\beta,\gamma\in\mathbb{R}$.
- $(1+\alpha)^\nu\geq 1+\nu\cdot\alpha$, $\alpha\geq -1$ $\forall\nu\in\mathbb{R}$. (Bernoulli)

Απόλυτη τιμή.

- $|\alpha|=\begin{cases}\alpha\ ,\ \alpha\nu\ \alpha\geq 0\\ -\alpha,\ \alpha\nu\ \alpha<0\end{cases}$
- $|\alpha|\geq 0\ \ \forall\alpha\in\mathbb{R}$.
- $|\alpha|^2=\alpha^2\ \ \forall\alpha\in\mathbb{R}$.
- $-|\alpha|\leq\alpha\leq|\alpha|\ \ \forall\alpha\in\mathbb{R}$.
- $|x|=\alpha\Leftrightarrow x=\alpha$ ή $x=-\alpha$.
- $|x|\leq\varepsilon\Leftrightarrow -\varepsilon\leq x\leq\varepsilon$.
- $|x|\geq\alpha\Leftrightarrow$ ή $x\leq-\alpha$ ή $x\geq\alpha$.
- $|\alpha\cdot\beta|=|\alpha|\cdot|\beta|\ \ \forall\alpha,\beta\in\mathbb{R}$.
- $\left|\frac{\alpha}{\beta}\right|=\frac{|\alpha|}{|\beta|}$, $\forall\alpha\in\mathbb{R}$, $\forall\beta\in\mathbb{R}^*$.
- $\big||\alpha|-|\beta|\big|\leq|\alpha\pm\beta|\leq|\alpha|+|\beta|$, $\forall\alpha,\beta\in\mathbb{R}$.

Δευτεροβάθμιο Τριώνυμο.

- Τριώνυμο $\pi(x)=\alpha x^2+\beta x+\gamma$, $\alpha\neq 0$.
- Διακρίνουσα $\Delta=\beta^2-4\cdot\alpha\cdot\gamma$.
- Ρίζες $x_{1,2}=\dfrac{-\beta\pm\sqrt{\Delta}}{2\cdot\alpha}$.
- $S=x_1+x_2=-\dfrac{\beta}{\alpha}$ και $P=x_1\cdot x_2=\dfrac{\gamma}{\alpha}$.

Αριθμητική Πρόοδος.

- Ορισμός $\alpha_{\nu+1}=\alpha_\nu+\omega$, $\nu=1,2,3,...$ ω: διαφορά.
- Νιοστός όρος $\alpha_\nu=\alpha_1+(\nu-1)\cdot\omega$, $\nu\in\mathbb{N}^*$.
- Άθροισμα ν πρώτων όρων $\Sigma_\nu=\dfrac{\alpha_1+\alpha_2}{2}\cdot\nu=\dfrac{2\alpha_1+(\nu-1)\cdot\omega}{2}\cdot\nu$
- β : αριθμητικός μέσος των $\alpha,\gamma \Leftrightarrow 2\beta=\alpha+\gamma$.

Γεωμετρική Πρόοδος.

- Ορισμός $\alpha_{\nu+1}=\alpha_\nu\cdot\lambda$, $\nu=1,2,3,...$ λ: λόγος.
- Νιοστός όρος $\alpha_\nu=\alpha_1\lambda^{\nu-1}$, $\nu\in\mathbb{N}^*$.
- Άθροισμα ν πρώτων όρων

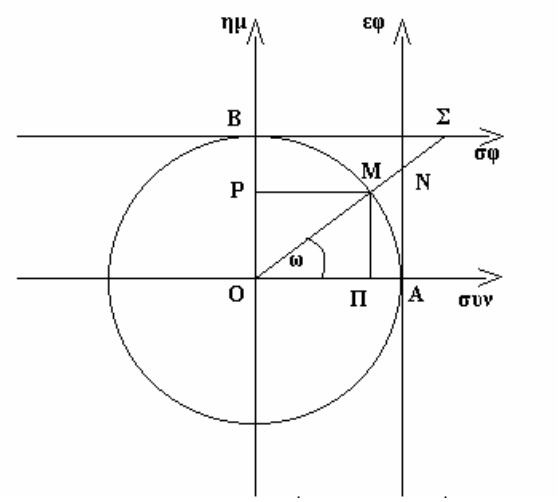
$$\Sigma_\nu=\begin{cases}\dfrac{\alpha_\nu\lambda-\alpha_1}{\lambda-1}=\dfrac{\alpha_1\left(\lambda^\nu-1\right)}{\lambda-1} & \text{αν } \lambda\neq 1\\ \nu\cdot\alpha_1 & \text{αν } \lambda=1\end{cases}$$

- Άθροισμα άπειρων όρων (αν $|\lambda|<1$) $\Sigma_\infty=\dfrac{\alpha_1}{1-\lambda}$
- β : γεωμετρικός μέσος των $\alpha,\gamma \Leftrightarrow \beta^2=\alpha\cdot\gamma$.

Αρμονική Πρόοδος.

- Ορισμός $(\alpha_{\nu+1})^{-1}=(\alpha_\nu)^{-1}+\omega$.
- β : αρμονικός μέσος των $\alpha,\gamma \Leftrightarrow \beta=\dfrac{2\alpha\gamma}{\alpha+\gamma}$

- Τριγωνομετρικός κύκλος - Τριγωνομετρικοί αριθμοί γωνίας.



$\text{συνω}=\overline{\text{ΟΠ}}$, $\text{ημω}=\overline{\text{ΟΡ}}$, $\text{εφω}=\overline{\text{ΑΝ}}$, $\text{σφω}=\overline{\text{ΒΣ}}$.

- Μετατροπή μονάδων $\frac{\mu}{180}=\frac{\alpha}{\pi}=\frac{\beta}{200}$.
- Πρόσημο τριγωνομετρικών αριθμών

Τεταρτημόριο	ημ	συν	εφ	σφ
1^o	+	+	+	+
2^o	+	–	–	–
3^o	–	–	+	+
4^o	–	+	–	–

- Τριγωνομετρικοί αριθμοί βασικών τόξων.

x	**0**	**π/6**	**π/4**	**π/3**	**π/2**	**π**	**3π/2**	**2π**
	0°	**30°**	**45°**	**60°**	**90°**	**180°**	**270°**	**360°**
Ημx	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	-1	0
συνx	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0	-1	0	1
Εφx	0	$\sqrt{3}/3$	1	$\sqrt{3}$	∞	0	∞	0
Σφx	∞	$\sqrt{3}$	1	$\sqrt{3}/3$	0	∞	0	∞

- Αναγωγή στο πρώτο τεταρτημόριο.

x	**-θ**	**(π/2)+θ**	**(π/2)-θ**	**π+θ**	**(3π/2)+θ**	**2π+θ**
ημx	-ημθ	συνθ	συνθ	-ημθ	-συνθ	ημθ
συνx	συνθ	-ημθ	ημθ	-συνθ	ημθ	συνθ
εφx	-εφθ	-σφθ	σφθ	εφθ	-σφθ	εφθ
σφx	-σφθ	-εφθ	εφθ	σφθ	-εφθ	σφθ

- $\eta\mu^2 x+\sigma\upsilon\nu^2 x=1$.
- $\varepsilon\varphi x=\dfrac{\eta\mu x}{\sigma\upsilon\nu x}$.
- $\sigma\varphi x=\dfrac{\sigma\upsilon\nu x}{\eta\mu x}$.
- $\varepsilon\varphi x\cdot\sigma\varphi x=1$.

- $\eta\mu(\alpha+\beta)=\eta\mu\alpha\cdot\sigma\upsilon\nu\beta+\sigma\upsilon\nu\alpha\cdot\eta\mu\beta$.
- $\sigma\upsilon\nu(\alpha+\beta)=\sigma\upsilon\nu\alpha\cdot\sigma\upsilon\nu\beta-\eta\mu\alpha\cdot\eta\mu\beta$.
- $\varepsilon\varphi(\alpha+\beta)=\dfrac{\varepsilon\varphi\alpha+\varepsilon\varphi\beta}{1-\varepsilon\varphi\alpha\cdot\varepsilon\varphi\beta}$
- $\sigma\varphi(\alpha+\beta)=\dfrac{\sigma\varphi\alpha\cdot\sigma\varphi\beta-1}{\sigma\varphi\alpha+\sigma\varphi\beta}$

- $\eta\mu(\alpha-\beta)=\eta\mu\alpha\cdot\sigma\upsilon\nu\beta-\sigma\upsilon\nu\alpha\cdot\eta\mu\beta$.
- $\sigma\upsilon\nu(\alpha-\beta)=\sigma\upsilon\nu\alpha\cdot\sigma\upsilon\nu\beta+\eta\mu\alpha\cdot\eta\mu\beta$.
- $\varepsilon\varphi(\alpha-\beta)=\dfrac{\varepsilon\varphi\alpha-\varepsilon\varphi\beta}{1+\varepsilon\varphi\alpha\cdot\varepsilon\varphi\beta}$
- $\sigma\varphi(\alpha-\beta)=\dfrac{\sigma\varphi\alpha\cdot\sigma\varphi\beta+1}{\sigma\varphi\alpha-\sigma\varphi\beta}$

- $\eta\mu 2\alpha=2\eta\mu\alpha\cdot\sigma\upsilon\nu\alpha = \dfrac{2\varepsilon\varphi\alpha}{1+\varepsilon\varphi^2\alpha}$.
- $\sigma\upsilon\nu 2\alpha=\begin{cases}\sigma\upsilon\nu^2\alpha-\eta\mu^2\alpha \\ 2\sigma\upsilon\nu^2\alpha-1 \\ 1-2\eta\mu^2\alpha\end{cases} = \dfrac{1-\varepsilon\varphi^2\alpha}{1+\varepsilon\varphi^2\alpha}$
- $\varepsilon\varphi 2\alpha=\dfrac{2\varepsilon\varphi\alpha}{1-\varepsilon\varphi^2\alpha}$
- $\sigma\varphi 2\alpha=\dfrac{\sigma\varphi^2\alpha-1}{2\sigma\varphi\alpha} = \dfrac{1-\varepsilon\varphi^2\alpha}{2\varepsilon\varphi\alpha}$
- $\eta\mu\alpha=\pm\sqrt{\dfrac{1-\sigma\upsilon\nu 2\alpha}{2}} = \pm\sqrt{\dfrac{\varepsilon\varphi^2\alpha}{1+\varepsilon\varphi^2\alpha}}$
- $\sigma\upsilon\nu\alpha=\pm\sqrt{\dfrac{1+\sigma\upsilon\nu 2\alpha}{2}} = \pm\sqrt{\dfrac{1}{1+\varepsilon\varphi^2\alpha}}$
- $\varepsilon\varphi\alpha=\pm\sqrt{\dfrac{1-\sigma\upsilon\nu 2\alpha}{1+\sigma\upsilon\nu 2\alpha}}$

- $\eta\mu 3\alpha = 3\eta\mu\alpha - 4\eta\mu^3\alpha$
- $\sigma\upsilon\nu 3\alpha = 4\sigma\upsilon\nu^3\alpha - 3\sigma\upsilon\nu\alpha$
- $\varepsilon\varphi 3\alpha = \dfrac{3\varepsilon\varphi\alpha - \varepsilon\varphi^3\alpha}{1 - 3\varepsilon\varphi^2\alpha}$
- $\sigma\varphi 3\alpha = \dfrac{\sigma\varphi^3\alpha - 3\sigma\varphi\alpha}{3\sigma\varphi^2\alpha - 1}$

- $2\eta\mu\alpha\cdot\sigma\upsilon\nu\beta = \eta\mu(\alpha+\beta) + \eta\mu(\alpha-\beta)$
- $2\sigma\upsilon\nu\alpha\cdot\sigma\upsilon\nu\beta = \sigma\upsilon\nu(\alpha+\beta) + \sigma\upsilon\nu(\alpha-\beta)$
- $2\eta\mu\alpha\cdot\eta\mu\beta = \sigma\upsilon\nu(\alpha-\beta) - \sigma\upsilon\nu(\alpha+\beta)$

- $\eta\mu\alpha + \eta\mu\beta = 2\cdot\eta\mu\dfrac{\alpha+\beta}{2}\cdot\sigma\upsilon\nu\dfrac{\alpha-\beta}{2}$
- $\eta\mu\alpha - \eta\mu\beta = 2\cdot\eta\mu\dfrac{\alpha-\beta}{2}\cdot\sigma\upsilon\nu\dfrac{\alpha+\beta}{2}$
- $\sigma\upsilon\nu\alpha + \sigma\upsilon\nu\beta = 2\cdot\sigma\upsilon\nu\dfrac{\alpha+\beta}{2}\cdot\sigma\upsilon\nu\dfrac{\alpha-\beta}{2}$
- $\sigma\upsilon\nu\alpha - \sigma\upsilon\nu\beta = 2\cdot\eta\mu\dfrac{\alpha+\beta}{2}\cdot\eta\mu\dfrac{\alpha-\beta}{2}$

Τριγωνομετρία. Ταυτότητες για στοιχεία τριγώνου.

- $\varepsilon\varphi A + \varepsilon\varphi B + \varepsilon\varphi\Gamma = \varepsilon\varphi A\cdot\varepsilon\varphi B\cdot\varepsilon\varphi\Gamma$.
- $\eta\mu A + \eta\mu B + \eta\mu\Gamma = 4\sigma\upsilon\nu\dfrac{A}{2}\cdot\sigma\upsilon\nu\dfrac{B}{2}\cdot\sigma\upsilon\nu\dfrac{\Gamma}{2}$.
- $\sigma\upsilon\nu A + \sigma\upsilon\nu B + \sigma\upsilon\nu\Gamma = 1 + 4\cdot\eta\mu\dfrac{A}{2}\cdot\eta\mu\dfrac{B}{2}\cdot\eta\mu\dfrac{\Gamma}{2}$.
- $\eta\mu 2A + \eta\mu 2B + \eta\mu 2\Gamma = 4\cdot\ \eta\mu A\cdot\eta\mu B\cdot\eta\mu\Gamma$
- $\sigma\upsilon\nu 2A + \sigma\upsilon\nu 2B + \sigma\upsilon\nu 2\Gamma = 1 - 4\cdot\ \sigma\upsilon\nu A\cdot\sigma\upsilon\nu B\cdot\sigma\upsilon\nu\Gamma$.
- $\sigma\varphi\dfrac{A}{2} + \sigma\varphi\dfrac{B}{2} + \sigma\varphi\dfrac{\Gamma}{2} = \sigma\varphi\dfrac{A}{2}\cdot\sigma\varphi\dfrac{B}{2}\cdot\sigma\varphi\dfrac{\Gamma}{2}$
- $\sigma\varphi A\cdot\sigma\varphi B + \sigma\varphi B\cdot\sigma\varphi\Gamma + \sigma\varphi\Gamma\cdot\sigma\varphi A = 1$.
- $\varepsilon\varphi\dfrac{A}{2}\cdot\varepsilon\varphi\dfrac{B}{2} + \varepsilon\varphi\dfrac{B}{2}\cdot\varepsilon\varphi\dfrac{\Gamma}{2} + \varepsilon\varphi\dfrac{\Gamma}{2}\cdot\varepsilon\varphi\dfrac{A}{2} = 1$.
- $\dfrac{\alpha}{\eta\mu A} = \dfrac{\beta}{\eta\mu B} = \dfrac{\gamma}{\eta\mu\Gamma} = 2R$. (Νόμος ημιτόνων.)
- $\alpha^2 = \beta^2 + \gamma^2 - 2\beta\gamma\cdot\sigma\upsilon\nu A$, $\beta^2 = \gamma^2 + \alpha^2 - 2\gamma\alpha\cdot\sigma\upsilon\nu B$, $\gamma^2 = \alpha^2 + \beta^2 - 2\alpha\beta\cdot\sigma\upsilon\nu\Gamma$
(Νόμος συνημιτόνων.)

Τριγωνομετρία. Τριγωνομετρικές εξισώσεις.

- $\eta\mu x=\eta\mu\alpha \Leftrightarrow x=2\kappa\pi+\alpha$ ή $x=(2\kappa+1)\pi-\alpha$, $\kappa\in\mathbf{Z}$.
- $\sigma\upsilon\nu x=\sigma\upsilon\nu\alpha \Leftrightarrow x=2\kappa\pi\pm\alpha$, $\kappa\in\mathbf{Z}$.
- $\varepsilon\varphi x=\varepsilon\varphi\alpha \Leftrightarrow x=\kappa\pi+\alpha$, $\kappa\in\mathbf{Z}$.
- $\sigma\varphi x=\sigma\varphi\alpha \Leftrightarrow x=\kappa\pi+\alpha$, $\kappa\in\mathbf{Z}$.

Λογάριθμοι.

- $\log_a(x\cdot y)=\log_a x+\log_a y$, $\forall x>0$, $\forall y>0$.
- $\log_a(x:y)=\log_a x-\log_a y$, $\forall x>0$, $\forall y>0$.
- $\log_a(x^\nu)=\nu\cdot\log_a x$, $\forall x>0$ και $\nu\in\mathbf{N}$.
- $\log x=\log_{10} x$.
- $\ln x=\log_e x$
- $\log_a x=\dfrac{\log_b x}{\log_b a}$

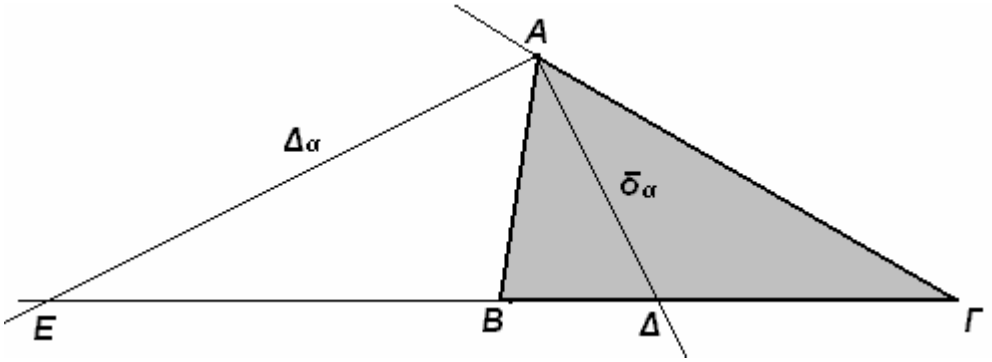
Συνδυαστική.

- Μεταθέσεις των ν στοιχείων : $M_\nu=\nu!$.
- Διατάξεις των μ στοιχείων σε ν θέσεις : $\Delta_\nu^\mu=\dfrac{\mu!}{(\mu-\nu)!}$
- Διατάξεις με επανάληψη των μ στοιχείων σε ν θέσεις : $E_\nu^\mu=\mu^\nu$.
- Συνδυασμοί των ν στοιχείων ανά κ : $\binom{\nu}{\kappa}=\dfrac{\nu!}{\kappa!(\nu-\kappa)!}$

Πιθανότητες.

- $P(A)=\dfrac{N(A)}{N(\Omega)}$.
  - $0\le P(A)\le 1$.
  - $P(\Omega)=1$.
  - $P(\varnothing)=0$.
  - $A\subseteq B \Rightarrow P(A)\le P(B)$
- $P(A\cup B)=P(A)+P(B)-P(A\cap B)$.
- $P(A')=1-P(A)$.
- $P(B|A)=\dfrac{P(A\cap B)}{P(A)}$.

Γεωμετρία **Θεωρήματα διχοτόμων.**

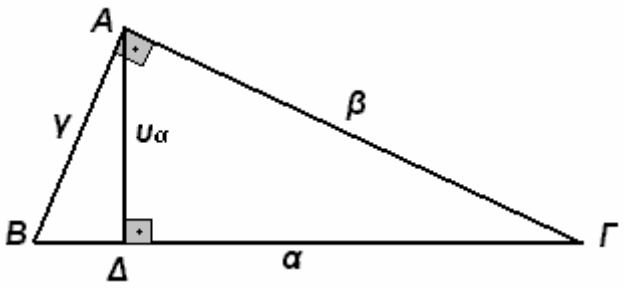


- $\frac{\Delta B}{\Delta \Gamma}=\frac{EB}{E\Gamma}=\frac{AB}{A\Gamma}$

- $B\Delta=\frac{\alpha\gamma}{\beta+\gamma}$
- $\Delta\Gamma=\frac{\alpha\beta}{\beta+\gamma}$
- $BE=\frac{\alpha\gamma}{\beta-\gamma}$
- $E\Gamma=\frac{\alpha\beta}{\beta-\gamma}$

- Τα Ε και Δ λέγονται αρμονικά συζυγή των Β και Γ.
- Τα Α,Β,Γ, και Δ λέγονται αρμονική τετράδα.

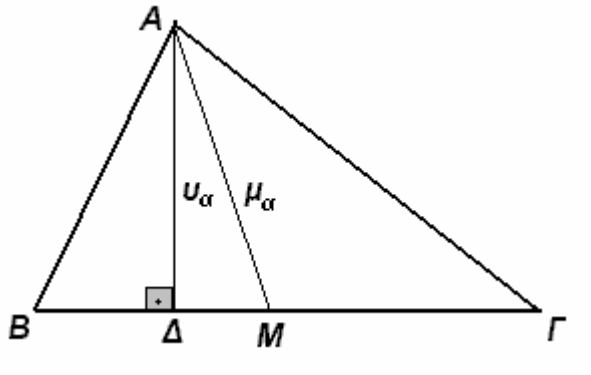
Γεωμετρία **Μετρικές σχέσεις σε ορθογώνια τρίγωνα.**



Τα τρίγωνα ΑΒΓ, ΔΒΑ και ΔΑΓ είναι όμοια.

- $\gamma^2=\alpha\cdot B\Delta$ και $\beta^2=\alpha\cdot\Gamma\Delta$.
- $\alpha^2=\beta^2+\gamma^2$. (Πυθαγόρειο Θεώρημα.)
- $\upsilon_\alpha=B\Delta\cdot\Delta\Gamma$.
- $\beta\cdot\gamma=\alpha\cdot\upsilon_\alpha$.
- $\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{1}{\upsilon_\alpha^2}$

Γεωμετρία. **Μετρικές σχέσεις σε τυχαίο τρίγωνο.**

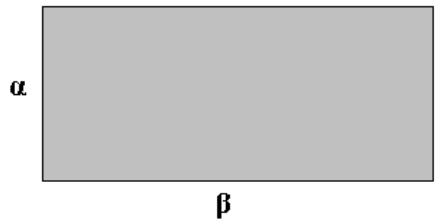


- $A\Gamma^2=AB^2+B\Gamma^2-2\cdot B\Gamma\cdot B\Delta$, αν η γωνία $\hat{B}$ είναι οξεία..
- $A\Gamma^2=AB^2+B\Gamma^2+2\cdot B\Gamma\cdot B\Delta$, αν η γωνία $\hat{B}$ είναι αμβλεία..
- $B\Delta=\dfrac{\gamma^2+\alpha^2-\beta^2}{2\cdot\alpha}$
- $\upsilon_\alpha=\dfrac{2\cdot\sqrt{\tau\cdot(\tau-\alpha)\cdot(\tau-\beta)\cdot(\tau-\gamma)}}{\alpha}$.
- $\beta^2+\gamma^2=2\cdot\mu_\alpha{}^2+\dfrac{\alpha^2}{2}$. ($1^o$ Θεώρημα διαμέσων.)
- $\beta^2+\gamma^2=2\cdot\alpha\cdot M\Delta$.
- $\mu_\alpha{}^2=\dfrac{2\beta^2+2\gamma^2-\alpha^2}{4}$.
- $\dfrac{1}{\upsilon_\alpha}+\dfrac{1}{\upsilon_\beta}+\dfrac{1}{\upsilon_\gamma}=\dfrac{1}{\rho}$.

Γεωμετρία **Κανονικά πολύγωνα.**

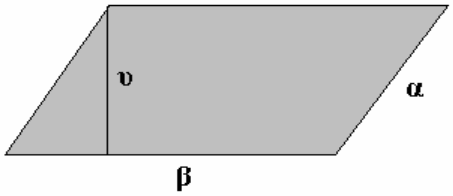
• $\omega_\nu=\dfrac{360^\circ}{\nu}$	• $\varphi_\nu=180^o-\omega_\nu$	• $\left(\dfrac{\lambda_\nu}{2}\right)+\alpha_\nu^2=R^2$	• $E_\nu=\dfrac{\nu}{2}\lambda_\nu\alpha_\nu$
• $\lambda_{2\nu}=\sqrt{2R^2-R\sqrt{4R^2-\lambda_\nu^2}}$		• $\lambda_3=R\sqrt{3}$	• $\alpha_3=\dfrac{R}{2}$
• $\lambda_4=R\sqrt{2}$	• $\alpha_4=\dfrac{R\sqrt{2}}{2}$	• $\lambda_6=R$	• $\alpha_6=\dfrac{R\sqrt{3}}{2}$
•	•	•	•

- Ορθογώνιο.



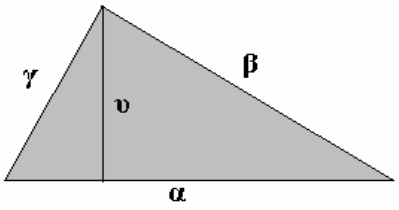
$E=\alpha\cdot\beta$

- Παραλληλόγραμμο.



$E=\beta\cdot\upsilon$

- Τρίγωνο.



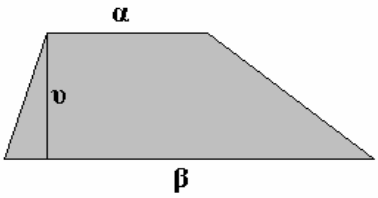
$$E=\frac{\beta\cdot\upsilon}{2}$$

$$E=\sqrt{\tau\cdot(\tau-\alpha)\cdot(\tau-\beta)\cdot(\tau-\gamma)}$$

$$E=\tau\cdot\rho$$

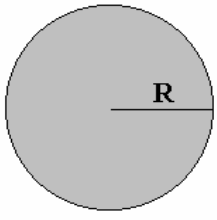
$$E=\frac{\alpha\cdot\beta\cdot\gamma}{4R}$$

- Τραπέζιο.



$$E=\frac{\alpha+\beta}{2}\cdot\upsilon$$

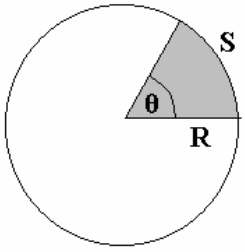
- Κύκλος.



$E=\pi R^2$

$\Gamma=2\pi R$

- Κυκλικός τομέας - τόξο.



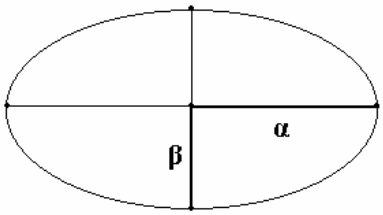
$S=\theta\cdot R$, θ ακτίνια.

$S=\frac{\theta}{360}2\pi R$, θ μοίρες.

$E=\frac{1}{2}\theta R^2$, θ ακτίνια.

$E=\frac{\theta}{360}\pi R^2$, θ μοίρες.

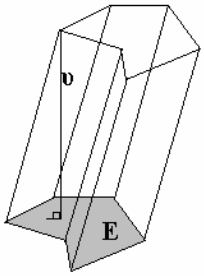
- Έλλειψη.



$E=\pi\cdot\alpha\cdot\beta$.

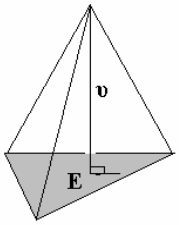
$\Gamma\approx\sqrt{\frac{1}{2}\left(\alpha^2+\beta^2\right)}$

- Πρίσμα.



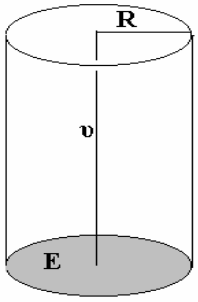
$V=E_\beta\cdot\upsilon$.

- Πυραμίδα



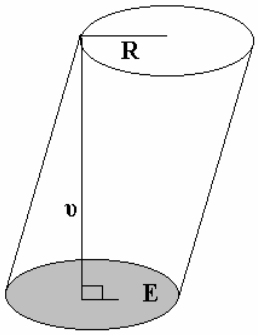
$V=\frac{1}{3}\cdot E_\beta\cdot\upsilon$.

- Κύλινδρος.



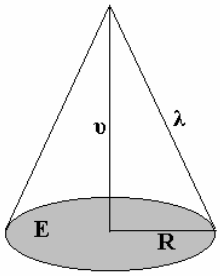
$E_\pi=2\pi R\cdot\upsilon$.
$E_{ολ}=2\pi R(R+\upsilon)$.
$V=\pi R^2\upsilon$.

- Πλάγιος κύλινδρος



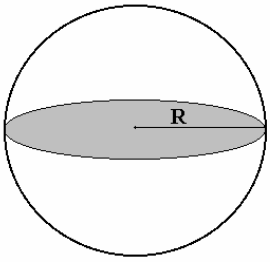
$V=\pi R^2 \cdot \upsilon$.

- Κώνος.



$V=\frac{1}{3} \cdot \pi R^2 \cdot \upsilon$.

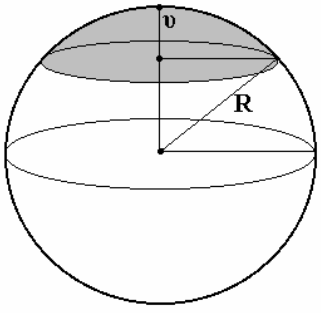
- Σφαίρα.



$E=4 \cdot \pi R^2$.

$V=\frac{4}{3} \cdot \pi R^3$.

- Κυκλικό τμήμα.



$E=2\pi R\upsilon$.

$V=\frac{1}{3} \cdot \pi \upsilon^2(3R-\upsilon)$.

Ανάλυση **Όρια.**

Όταν υπάρχουν τα όρια των συναρτήσεων f και g τότε ισχύουν

- $\lim\limits_{x\to\sigma} f(x)=\ell \Leftrightarrow$
  - $\Leftrightarrow \lim\limits_{x\to\sigma}[f(x)-\ell]=0 \Leftrightarrow$
  - $\Leftrightarrow \lim\limits_{x\to\sigma}(-f(x))=-\ell \Leftrightarrow$
- $\lim\limits_{x\to\sigma} f(x)=\ell \Rightarrow$
  $\Rightarrow \dfrac{|\ell|}{2}<|f(x)|<\dfrac{3|\ell|}{2}$
- $\lim\limits_{x\to\sigma}[\lambda\cdot f(x)]=\lambda\cdot\lim\limits_{x\to\sigma} f(x)$

- $\lim\limits_{x\to\sigma}(f(x)+g(x))=\lim\limits_{x\to\sigma} f(x)+\lim\limits_{x\to\sigma} g(x)$
- $\lim\limits_{x\to\sigma}(f(x)-g(x))=\lim\limits_{x\to\sigma} f(x)-\lim\limits_{x\to\sigma} g(x)$
- $\lim\limits_{x\to\sigma}(f(x)\cdot g(x))=\lim\limits_{x\to\sigma} f(x)\cdot\lim\limits_{x\to\sigma} g(x)$
- $\lim\limits_{x\to\sigma}\dfrac{f(x)}{g(x)}=\dfrac{\lim\limits_{x\to\sigma} f(x)}{\lim\limits_{x\to\sigma} g(x)}$ { Εφόσον ορίζονται καλώς τα κλάσματα..
- $\lim\limits_{x\to\sigma}\sqrt[\nu]{f(x)}=\sqrt[\nu]{\lim\limits_{x\to\sigma} f(x)}$ { Εφόσον ορίζονται καλώς οι ρίζες.

Ανάλυση **Παράγωγοι.**

- $[c]'=0$
- $[x]'=1$
- $[x^\nu]'=\nu x^{\nu-1}$
- $[x^\rho]'=\rho x^{\rho-1}$
- $[\eta\mu x]'=\sigma\upsilon\nu x$
- $[\sigma\upsilon\nu x]'=-\eta\mu x$
- $[e^x]'=e^x$
- $[\ln x]'=\dfrac{1}{x}$
- $[\varepsilon\varphi x]'=\dfrac{1}{\sigma\upsilon\nu^2 x}$
- $[\sigma\varphi x]'=\dfrac{-1}{\eta\mu^2 x}$
- $[f+g]'=f'+g'$
- $[f-g]'=f'-g'$
- $[f\cdot g]'=f'\cdot g+f\cdot g'$
- $\left[\dfrac{f}{g}\right]'=\dfrac{f'\cdot g-f\cdot g'}{g^2}$
- $[\lambda\cdot f]'=\lambda\cdot f'$
- $[f^\nu]'=\nu\cdot f^{\nu-1}\cdot f'$

- $[f(g(x))]'=f'(g(x))\cdot g'(x)$ ή $\dfrac{df(g(x))}{dx}=\dfrac{df(g(x))}{dg(x)}\cdot\dfrac{dg(x)}{dx}$

- $\int_\alpha^\beta f(x)dx := \lim\left[\frac{\beta-\alpha}{\nu}\cdot\sum_{\kappa=1}^{\nu} f\left(\alpha+\kappa\cdot\frac{\beta-\alpha}{\nu}\right)\right]$
- $\int_\alpha^\beta [f(x)+g(x)]dx = \int_\alpha^\beta f(x)dx + \int_\alpha^\beta g(x)dx$
- $\int_\alpha^\beta \lambda\cdot f(x)dx = \lambda\cdot\int_\alpha^\beta f(x)dx$
- $\int_\alpha^\beta f(x)dx = \int_\alpha^\gamma f(x)dx + \int_\gamma^\beta f(x)dx$
- $\min f\cdot(\beta-\alpha) \le \int_\alpha^\beta f(x)dx \le \max f\cdot(\beta-\alpha)$

- $\int_\alpha^\alpha f(x)dx = 0$
- $\int_\beta^\alpha f(x)dx = -\int_\alpha^\beta f(x)dx$
- $\int_\alpha^\beta f'(x)dx = f(\beta)-f(\alpha)$
- $\left[\int_\alpha^x f(t)dt\right]' = f(x)$
- $\left[\int_\alpha^{g(x)} f(t)dt\right]' = f(g(x))\cdot g'(x)$

- $\int_\alpha^\beta f'(x)\cdot g(x)dx = \left[f(x)\cdot g(x)\right]_\alpha^\beta - \int_\alpha^\beta f(x)\cdot g'(x)dx$
- $\int_\alpha^\beta f(g(x))\cdot g'(x)dx = \int_{g(\alpha)}^{g(\beta)} f(y)dy$
- $\bar{f} = \frac{1}{\beta-\alpha}\cdot\int_\alpha^\beta f(x)dx$

- $\int f'(x)dx = f(x)+c$
- $\int 1dx = x+c$
- $\int \frac{1}{x}dx = \ln|x|+c$
- $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1}+c$

- $\int \sigma\upsilon\nu x dx = \eta\mu x + c$
- $\int \eta\mu x dx = -\sigma\upsilon\nu x + c$
- $\int \frac{1}{\sigma\upsilon\nu^2 x}dx = \varepsilon\varphi x + c$
- $\int \frac{1}{\eta\mu^2 x}dx = -\sigma\varphi x + c$

- $\int e^x dx = e^x + c$
- $\int \alpha^x dx = \frac{\alpha^x}{\ln\alpha}+c$

ΠΙΝΑΚΕΣ ΤΡΙΓΩΝΟΜΕΤΡΙΚΩΝ ΑΡΙΘΜΩΝ

ω	ημω	συνω	εφω	σφω
0 º	0,0000000	1,0000000	0,0000000	
1 º	0,0174524	0,9998477	0,0174551	57,2899616
2 º	0,0348995	0,9993908	0,0349208	28,6362533
3 º	0,0523360	0,9986295	0,0524078	19,0811367
4 º	0,0697565	0,9975641	0,0699268	14,3006663
5 º	0,0871557	0,9961947	0,0874887	11,4300523
6 º	0,1045285	0,9945219	0,1051042	9,5143645
7 º	0,1218693	0,9925462	0,1227846	8,1443464
8 º	0,1391731	0,9902681	0,1405408	7,1153697
9 º	0,1564345	0,9876883	0,1583844	6,3137515
10 º	0,1736482	0,9848078	0,1763270	5,6712818
11 º	0,1908090	0,9816272	0,1943803	5,1445540
12 º	0,2079117	0,9781476	0,2125566	4,7046301
13 º	0,2249511	0,9743701	0,2308682	4,3314759
14 º	0,2419219	0,9702957	0,2493280	4,0107809
15 º	0,2588190	0,9659258	0,2679492	3,7320508
16 º	0,2756374	0,9612617	0,2867454	3,4874144
17 º	0,2923717	0,9563048	0,3057307	3,2708526
18 º	0,3090170	0,9510565	0,3249197	3,0776835
19 º	0,3255682	0,9455186	0,3443276	2,9042109
20 º	0,3420201	0,9396926	0,3639702	2,7474774
21 º	0,3583679	0,9335804	0,3838640	2,6050891
22 º	0,3746066	0,9271839	0,4040262	2,4750869
23 º	0,3907311	0,9205049	0,4244748	2,3558524
24 º	0,4067366	0,9135455	0,4452287	2,2460368
25 º	0,4226183	0,9063078	0,4663077	2,1445069
26 º	0,4383711	0,8987940	0,4877326	2,0503038
27 º	0,4539905	0,8910065	0,5095254	1,9626105
28 º	0,4694716	0,8829476	0,5317094	1,8807265
29 º	0,4848096	0,8746197	0,5543091	1,8040478
30 º	0,5000000	0,8660254	0,5773503	1,7320508
31 º	0,5150381	0,8571673	0,6008606	1,6642795
32 º	0,5299193	0,8480481	0,6248694	1,6003345
33 º	0,5446390	0,8386706	0,6494076	1,5398650
34 º	0,5591929	0,8290376	0,6745085	1,4825610
35 º	0,5735764	0,8191520	0,7002075	1,4281480
36 º	0,5877853	0,8090170	0,7265425	1,3763819
37 º	0,6018150	0,7986355	0,7535541	1,3270448
38 º	0,6156615	0,7880108	0,7812856	1,2799416
39 º	0,6293204	0,7771460	0,8097840	1,2348972
40 º	0,6427876	0,7660444	0,8390996	1,1917536
41 º	0,6560590	0,7547096	0,8692867	1,1503684
42 º	0,6691306	0,7431448	0,9004040	1,1106125
43 º	0,6819984	0,7313537	0,9325151	1,0723687
44 º	0,6946584	0,7193398	0,9656888	1,0355303
45 º	0,7071068	0,7071068	1,0000000	1,0000000

ω	ημω	συνω	εφω	σφω
45 º	0,7071068	0,7071068	1,0000000	1,0000000
46 º	0,7193398	0,6946584	1,0355303	0,9656888
47 º	0,7313537	0,6819984	1,0723687	0,9325151
48 º	0,7431448	0,6691306	1,1106125	0,9004040
49 º	0,7547096	0,6560590	1,1503684	0,8692867
50 º	0,7660444	0,6427876	1,1917536	0,8390996
51 º	0,7771460	0,6293204	1,2348972	0,8097840
52 º	0,7880108	0,6156615	1,2799416	0,7812856
53 º	0,7986355	0,6018150	1,3270448	0,7535541
54 º	0,8090170	0,5877853	1,3763819	0,7265425
55 º	0,8191520	0,5735764	1,4281480	0,7002075
56 º	0,8290376	0,5591929	1,4825610	0,6745085
57 º	0,8386706	0,5446390	1,5398650	0,6494076
58 º	0,8480481	0,5299193	1,6003345	0,6248694
59 º	0,8571673	0,5150381	1,6642795	0,6008606
60 º	0,8660254	0,5000000	1,7320508	0,5773503
61 º	0,8746197	0,4848096	1,8040478	0,5543091
62 º	0,8829476	0,4694716	1,8807265	0,5317094
63 º	0,8910065	0,4539905	1,9626105	0,5095254
64 º	0,8987940	0,4383711	2,0503038	0,4877326
65 º	0,9063078	0,4226183	2,1445069	0,4663077
66 º	0,9135455	0,4067366	2,2460368	0,4452287
67 º	0,9205049	0,3907311	2,3558524	0,4244748
68 º	0,9271839	0,3746066	2,4750869	0,4040262
69 º	0,9335804	0,3583679	2,6050891	0,3838640
70 º	0,9396926	0,3420201	2,7474774	0,3639702
71 º	0,9455186	0,3255682	2,9042109	0,3443276
72 º	0,9510565	0,3090170	3,0776835	0,3249197
73 º	0,9563048	0,2923717	3,2708526	0,3057307
74 º	0,9612617	0,2756374	3,4874144	0,2867454
75 º	0,9659258	0,2588190	3,7320508	0,2679492
76 º	0,9702957	0,2419219	4,0107809	0,2493280
77 º	0,9743701	0,2249511	4,3314759	0,2308682
78 º	0,9781476	0,2079117	4,7046301	0,2125566
79 º	0,9816272	0,1908090	5,1445540	0,1943803
80 º	0,9848078	0,1736482	5,6712818	0,1763270
81 º	0,9876883	0,1564345	6,3137515	0,1583844
82 º	0,9902681	0,1391731	7,1153697	0,1405408
83 º	0,9925462	0,1218693	8,1443464	0,1227846
84 º	0,9945219	0,1045285	9,5143645	0,1051042
85 º	0,9961947	0,0871557	11,4300523	0,0874887
86 º	0,9975641	0,0697565	14,3006663	0,0699268
87 º	0,9986295	0,0523360	19,0811367	0,0524078
88 º	0,9993908	0,0348995	28,6362533	0,0349208
89 º	0,9998477	0,0174524	57,2899616	0,0174551
90 º	1,0000000	0,0000000	############	0,0000000