Διαγώνισμα Νεκτάριου 2022

Θέμα Α

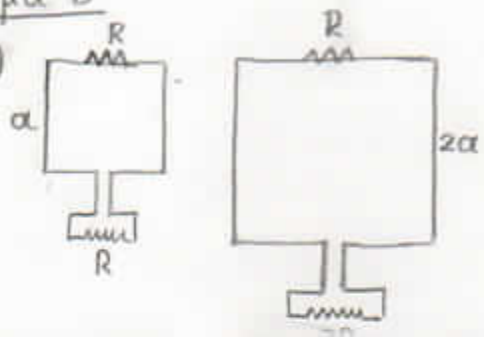
Α1 → α | Α2 → β | Α3 → γ | Α4 → β | Α5 → ΛΣΣΛΣ (θε γέει "κεντρική")

Α1) $\omega_{κεν} = \frac{\omega_1+\omega_2}{2} \Rightarrow 2\pi f_{κιν} = 2\pi \frac{f_1+f_2}{2} \Rightarrow T_{κιν} = \frac{2}{f_1+f_2}$ Μεγιστοποίηση $\frac{T_{κιν}}{2}$

Α2) $m_1 = m_2 \Rightarrow W_1 = W_2 \Rightarrow p_1 = p_2$

Θέμα Β

Β1)



$\bar{P}_1 = \bar{P}_2 \Rightarrow I_{εν,1}^2 \cdot R = I_{εν,2}^2 \cdot 3R$

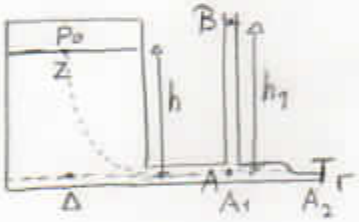
$\left(\frac{E_{εν,1}}{R_{ολ1}}\right)^2 = \left(\frac{E_{εν,2}}{R_{ολ2}}\right)^2 \cdot 3 \Rightarrow$

$\left(\frac{E_{εν,1}}{2R}\right)^2 = \left(\frac{E_{εν,2}}{4R}\right)^2 \cdot 3 \Rightarrow \frac{E_{εν,1}^2}{4R^2} = \frac{3E_{εν,2}^2}{16R^2} \Rightarrow$

$\frac{E_{εν,1}}{E_{εν,2}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{E_{max,1}/\sqrt{2}}{E_{max,2}/\sqrt{2}} = \frac{\sqrt{3}}{2} \Rightarrow \frac{N\omega_1 B a^2}{N\omega_2 B 4a^2} = \frac{\sqrt{3}}{2} \Rightarrow \frac{\omega_1}{\omega_2} = 2\sqrt{3} \Rightarrow$

$\frac{2\pi/T_1}{2\pi/T_2} = 2\sqrt{3} \Rightarrow \frac{T_2}{T_1} = 2\sqrt{3} \Rightarrow \frac{T_1}{T_2} = \frac{1}{2\sqrt{3}} = \frac{1\sqrt{3}}{2\sqrt{3}\sqrt{3}} = \frac{\sqrt{3}}{6}$ → (β)

Β2)



$A_1 = 3A_2$

Στρόφιγγα κλειστή

$P_A = P_Δ \Rightarrow p_{ατμ} + \rho g h_1 = p_o + \rho g h$

Στρόφιγγα ανοιχτή

ΖΑΓ ρευματική γραμμή:

$p_o + \rho g h + 0 = p_{ατμ} + \frac{1}{2}\rho v_2^2$

$A_1 v_1 = A_2 v_2 \Rightarrow v_2 = 3v_1$

$\rho g h_1 = \frac{1}{2}\rho v_2^2 \Rightarrow \boxed{g h_1 = \frac{v_2^2}{2}}$ (1)

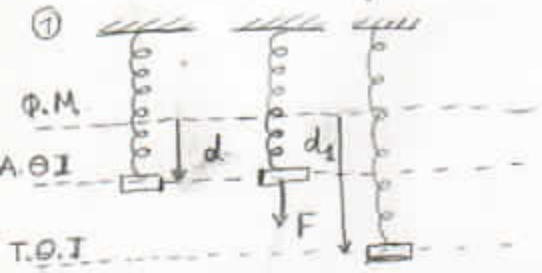
ΑΓ ρευμ/κη γραμμή: $p_A' + \frac{1}{2}\rho v_1^2 = p_{ατμ} + \frac{1}{2}\rho v_2^2 \Rightarrow p_{ατμ} + \rho g h_1' = p_{ατμ} +$

$+ \frac{1}{2}\rho\left(v_2^2 - \frac{v_2^2}{9}\right) \Rightarrow \rho g h_1' = \frac{1}{2}\rho \frac{8v_2^2}{9} \Rightarrow \boxed{g h_1' = \frac{4v_2^2}{9}}$ (2)

(1), (2) → $\frac{h_1'}{h_1} = \frac{4v_2^2/9}{v_2^2/2} = \frac{8v_2^2}{9v_2^2} = \frac{8}{9}$ → (δ)

B3) $k_1 = k_2 = k$, d, m

① $F = 2mg \rightarrow A_1$ ② $F = 2mg$ (κατώρι σε $x = d$)

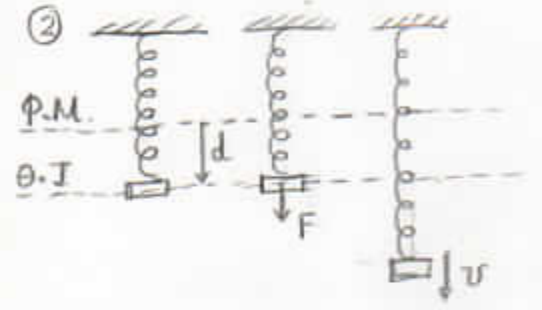


ΑΘΙ: $\Sigma F = 0 \Rightarrow k d = mg \Rightarrow d = mg/k$

ΤΘΙ $\Sigma F = 0 \Rightarrow k d_1 = mg + F \Rightarrow k d_1 = 3mg$

$\Rightarrow d_1 = 3mg/k \Rightarrow d_1 = 3d$

$A_1 = d_1 - d \Rightarrow A_1 = 2d$



$\Theta.I : \Sigma F = 0 \Rightarrow d = \frac{mg}{k}$

$E_{T2} = W_F = F \cdot d = 2mg \cdot mg/k = \frac{2m^2g^2}{k}$

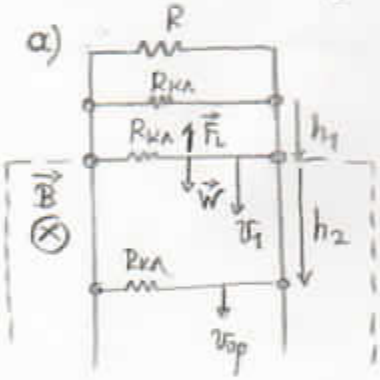
$E_{T2} = \frac{1}{2} k A_2^2 \Rightarrow \frac{2m^2g^2}{k} = \frac{1}{2} k A_2^2 \Rightarrow$

$A_2^2 = \frac{4m^2g^2}{k^2} \Rightarrow A_2 = \frac{2mg}{k} \Rightarrow A_2 = 2d$

Άρα $A_1 = A_2 \rightarrow$ (α)

Θέμα Γ | $m = 0,1\,kg$, $R_{KA} = 1\Omega$, $R = 3\Omega$, $h_1 = 0,25\,m$, $B = 2\,T$, $v_1 = 2\,m/s$

α)



ΘΜΚΕ: $\frac{1}{2} m v_1^2 - 0 = mg h_1 - T \cdot h_1 \Rightarrow \frac{1}{2} \cdot 0,1 \cdot 4 = 1 \cdot 0,25 - T \cdot 0,25$

$\Rightarrow 0,2 = 0,25 - 0,25T \Rightarrow 0,25T = 0,05 \Rightarrow T = 0,2N$

β) $\frac{dK}{dt} = -2,4\,J/s$

$E = Bv\ell$, $I = \frac{E}{R_{KA}+R} \Rightarrow I = \frac{B\ell}{R_{KA}+R} v$, $F_L = B\ell v \Rightarrow$

$F_L = \frac{B^2\ell^2}{R_{KA}+R} \cdot v$

Με την είσοδο $F_L = \frac{4 \cdot \ell^2}{4} \cdot 2 \Rightarrow F_L = 2\ell^2$

$\frac{dK}{dt} = P_{\Sigma F} = \Sigma F\, v_1 = (mg - F_L - T) \Rightarrow -2,4 = (0,8 - 2\ell^2) \cdot 2 \Rightarrow 0,8 - 2\ell^2 = -1,2 \Rightarrow$

$2\ell^2 = 2 \Rightarrow \ell = 1m$

γ) Όταν $v_2 = 0,5 v_1 = 1\,m/s$

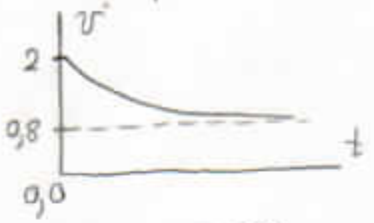
$I = \frac{B\ell}{R_{KA}+R} \cdot v_2 = \frac{2 \cdot 1 \cdot 1}{4} = 0,5A$, $P_\theta = I^2 \cdot R_{KA} = 0,25 \cdot 1 = 0,25W$

$E = B\ell v_1 = 2 \cdot 1 \cdot 1 = 2V$, $V_{KA} = E - I \cdot R_{KA} = 2 - 0,5 \cdot 1 = 1,5V$

β) $\Sigma F=0 \Rightarrow mg - F_L - T = 0 \Rightarrow F_L = mg - T \Rightarrow \frac{B^2\ell^2}{R_{ολ}} \cdot v_{op} = mg - T$

$\Rightarrow \frac{4\cdot 1^2}{4} \cdot v_{op} = 1 - 0,2 \Rightarrow v_{op} = 0,8\ m/s$

$P_{F_L} = -|F_L| \cdot v_{op} = -\frac{4\cdot 1^2 \cdot 0,8}{4} \cdot 0,8 = -0,64\ W$



ε) $Q_Σ = 0,568\ J$

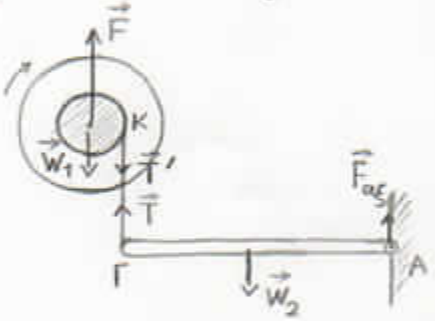
ΘΜΚΕ: $\frac{1}{2}mv_{op}^2 - \frac{1}{2}mv_1^2 = \overbrace{W_{F_L}}^{-Q} + W_{βαρ} + W_T \Rightarrow$

$\frac{1}{2}mv_{op}^2 - \frac{1}{2}mv_1^2 = -Q + mgh_2 - Th_2 \Rightarrow h_2 = 0,5m$

$|\Delta q| = \frac{|\Delta\Phi|}{R_{ολ}} = \frac{B\cdot\ell\cdot h_2}{R_{ολ}} = 0,25C$

Θέμα Δ $m=3kg$, $R=0,5m$, $r=R/2$, $M=6kg$, $L=2m$

α)



Δίσκος

$\alpha_K = \alpha_r = 0 \Rightarrow \alpha_{cm} - \alpha_{γων} r = 0 \Rightarrow \alpha_{cm} = \alpha_{γων} r$ ①

$F - T' - mg = m\alpha_{cm}$ ②

$I = \frac{1}{2}mR^2 = \frac{1}{2}\cdot 3\cdot 0,25 = \frac{3}{2}\cdot\frac{1}{4} = \frac{3}{8}\ kgm^2$

$Tr = \frac{1}{2}mR^2 \cdot \alpha_{γων} \overset{①}{\Rightarrow} T = \frac{1}{2}m\frac{R^2}{r^2}\cdot\alpha_{cm}$ ③

Ράβδος $W_2\frac{L}{2} - TL = 0 \Rightarrow T = \frac{Mg}{2} = 30N = T'$

$\frac{②}{③} \Rightarrow \frac{F-T'-mg}{T} = \frac{1}{\frac{1}{2}\frac{R^2}{r^2}} \Rightarrow \frac{F-30-30}{30} = \frac{1}{0,5\cdot 4} \Rightarrow \frac{F-60}{30} = \frac{1}{2} \Rightarrow 2F-120=30$

$\Rightarrow F = 75N$

β) ② $\Rightarrow 75-30-30 = 3\alpha_{cm} \Rightarrow 15 = 3\alpha_{cm} \Rightarrow \alpha_{cm} = 5\ m/s^2$, $\alpha_{γων} = \frac{5}{0,25} = 20\ rad/s^2$

Σε $t=1s$: $\Delta x_{cm} = \frac{1}{2}\alpha_{cm}t^2 = \frac{1}{2}\cdot 5\cdot 1^2 = 2,5m$, $v_{cm} = \alpha_{cm}t = 5\ m/s$

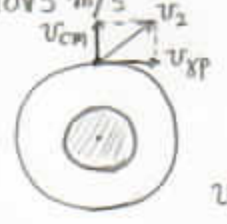
$\omega = \alpha_{γων} t = 20\ rad/s$

$W_F = F\cdot\Delta x_{cm} = 75\cdot 2,5 = 187,5\ J$

$K = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\cdot I\cdot\omega^2 = \frac{1}{2}\cdot 3\cdot 25 + \frac{1}{2}\cdot\frac{3}{8}\cdot 400 = 37,5 + 75 = 112,5\ J$

$Π = \frac{K}{W_F}\cdot 100 = 60\%$

γ) $v_2 = 10\sqrt{5}\ m/s$



$v_2^2 = v_{cm}^2 + v_{εφ}^2$

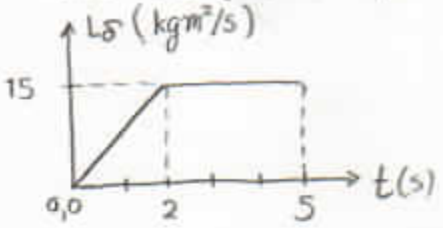
$v_{εφ} = \omega R$, $v_{cm} = \omega r$ $\Big\}$ $v_{εφ} = 2v_{cm}$ $\Big\}$ $v_2^2 = 5v_{cm}^2 \Rightarrow v_{cm} = \frac{v_2}{\sqrt{5}} = 10\ m/s$

$v_{cm} = \alpha_{cm}\cdot t_2 \Rightarrow t_2 = 2s$

$L_\delta = I\cdot\omega \Rightarrow L_\delta = I\cdot a_{\gamma\omega v}\cdot t \Rightarrow L_\delta = \frac{3}{8}\cdot 20\cdot t \Rightarrow L_\delta = \frac{60}{8}t$

$\Rightarrow L_\delta = \frac{15}{2}\cdot t \Rightarrow L_\delta = 7{,}5t$, $t\in[0,2s]$

Για $t\in(2s, 5s]$, $L_\delta = 7{,}5\cdot 2 = 15\ kgm^2/s$



δ) t_3 $\frac{dU_\theta}{dt} = 390\ J/s$

$\frac{dU_\theta}{dt} = mg\cdot v'_{cm} \Rightarrow 390 = 30\cdot v'_{cm} \Rightarrow v'_{cm} = 13\frac{m}{s}$

Για $t>t_2$ αλλάζει η επιτάχυνση

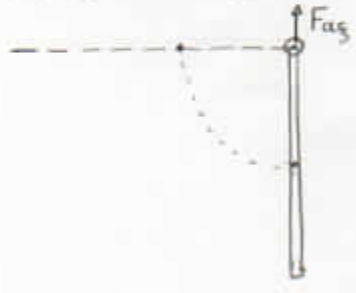
$F - mg = m\alpha'_{cm} \Rightarrow \alpha'_{cm} = \frac{75-30}{3} = \frac{45}{3} = 15\ m/s^2$

$v'_{cm} = v_{cm,0} + \alpha'_{cm}\cdot\Delta t \Rightarrow 13 = 10 + 15\cdot\Delta t \Rightarrow \Delta t = \frac{3}{15} = 0{,}2\ s$

Άρα $t_3 = 2{,}2\ s$ Στα 2s, η $\omega' = a_{\gamma\omega v}\cdot t_2 = 40\ rad/s$

$\Delta\theta = \omega\cdot\Delta t = 40\cdot 0{,}2 = 8\ rad$, $N = \frac{\Delta\theta}{2\pi} = \frac{4}{\pi}$ στροφές

$P_F(t_3) = F\ v'_{cm} = 75\cdot 13 = 975\ W$

ε) 

$\omega_\rho = max$, όταν $a_{\gamma\omega v} = 0$ → κατώτερη θέση

$Mg\frac{L}{2} = \frac{1}{2}\frac{1}{3}ML^2\omega_\rho^2 \Rightarrow \omega_\delta = \sqrt{\frac{3g}{L}} \Rightarrow$

$\omega_\delta = \sqrt{15}\ rad/s$

Δεν απαιτείται $\vec{\alpha}_\varepsilon$ Άρα $\vec{F}_{\alpha\xi}$ ακτινική ώστε

$\Sigma F_r = F_k \Rightarrow F_{\alpha\xi} - Mg = M\omega^2\cdot\frac{L}{2} \Rightarrow$

$\Rightarrow F_{\alpha\xi} - 60 = 6\cdot 15\cdot 1 \Rightarrow F_{\alpha\xi} = 150\ N$