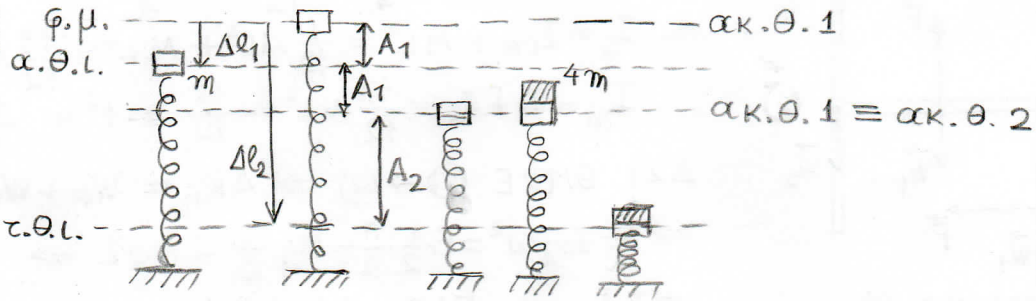
Απαντήσεις Γ' Θετικής – 2017

Θέμα Α

A1	A2	A3	A4	A5
α	β	β	γ	Λ Σ Σ Λ Λ

Θέμα Β

Β1



α.θ.ι. $\Sigma\vec{F}=0 \Rightarrow |\Delta\ell_1| = \frac{mg}{k} = A_1$ (2μ)

τ.θ.ι. $\Sigma\vec{F}=0 \Rightarrow |\Delta\ell_2| = \frac{4mg}{k} = 4A_1$ (2μ)

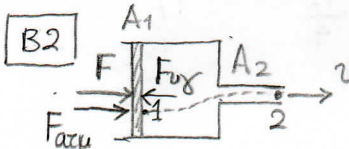
Από το σχήμα $A_2 = |\Delta\ell_2| - 2A_1 = 4A_1 - 2A_1 = 2A_1$ (2μ)

$\omega_1 = \sqrt{\frac{k}{m}}$, $\omega_2 = \sqrt{\frac{k}{4m}} = \frac{1}{2}\sqrt{\frac{k}{m}} = \frac{\omega_1}{2}$ (2μ)

$\frac{\upsilon_{max,1}}{\upsilon_{max,2}} = \frac{\omega_1 \cdot A_1}{\omega_2 \cdot A_2} = \frac{\omega_1 \cdot A_1}{\frac{\omega_1}{2} \cdot 2A_1} = 1 \Rightarrow \upsilon_{max,2} = \upsilon_{max,1} \rightarrow$ (α) (4μ)

(1μ)

Β2



Αφού το υγρό εξέρχεται με σταθερή ταχύτητα υ και το έμβολο θα κινείται με σταθερή ταχύτητα υ_ε

$\Sigma\vec{F}=0 \Rightarrow F_{ατ} + F - F_{αγ} = 0 \overset{:A_1}{\Rightarrow} p_1 = p_{ατ} + \frac{F}{A_1}$ ① (2μ)

1→2: $A_1\upsilon_\varepsilon = A_2\upsilon \Rightarrow \upsilon_\varepsilon = \frac{A_2}{A_1}\cdot\upsilon \overset{A_2 << A_1}{\Longrightarrow} \upsilon_\varepsilon \simeq 0$ ② (1μ)

1→2: $p_1 + \frac{1}{2}\rho\upsilon_\varepsilon^2 = p_2 + \frac{1}{2}\rho\upsilon^2 \overset{①}{\overset{②}{\Rightarrow}} p_{ατ} + \frac{F}{A_1} = p_{ατ} + \frac{1}{2}\rho\upsilon^2 \Rightarrow \upsilon = \sqrt{\frac{2F}{A_1\rho}}$ ③ (2μ)

Αν $F' = 2F$ τότε $\upsilon' = \sqrt{\frac{4F}{A_1\rho}}$ ④ (2μ)

$\frac{④}{③} \Rightarrow \frac{\upsilon'}{\upsilon} = \sqrt{2} \Rightarrow \upsilon' = \sqrt{2}\cdot\upsilon \rightarrow$ (β) (4μ)

(1μ)

Θέμα Γ

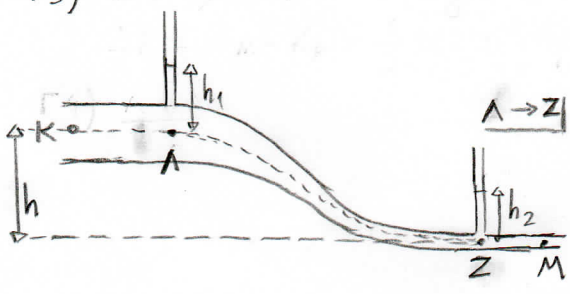
$\Pi = 3{,}5\cdot10^{-3}\ m^3/s$, $\rho = 10^3\ kg/m^3$, $h = 0{,}5\ m$, $A_1 = 7\cdot10^{-3}\ m^2$, $A_2 = 1\cdot10^{-3}\ m^2$, $h_1 = 0{,}8\ m$

Γ1) $\Pi = A_1\upsilon_1 \Rightarrow \upsilon_1 = \frac{3{,}5\cdot10^{-3}}{7\cdot10^{-3}} = 0{,}5\ m/s$, $\Pi = A_2\upsilon_2 \Rightarrow \upsilon_2 = \frac{3{,}5\cdot10^{-3}}{1\cdot10^{-3}} = 3{,}5\ m/s$ (6μ)

Γ2) $p_\Lambda = p_{ατ} + \rho g h_1 = 1\cdot10^5 + 0{,}8\cdot10^4 = 1{,}08\cdot10^5\ Pα$ (3μ)

Κ→Λ: $A_1\upsilon_K = A_1\upsilon_\Lambda \Rightarrow \upsilon_K = \upsilon_\Lambda$ και $p_K + \frac{1}{2}\rho\upsilon_K^2 = p_\Lambda + \frac{1}{2}\rho\upsilon_\Lambda^2 \Rightarrow p_\Lambda = 1{,}08\cdot10^5\ Pα$ (2μ) (4μ)

Γ3)



Λ→Ζ: $p_\Lambda + \rho g h + \frac{1}{2}\rho\upsilon_1^2 = p_Z + \frac{1}{2}\rho\upsilon_2^2 \Rightarrow p_Z = p_\Lambda + \rho g h + \frac{1}{2}\rho(\upsilon_1^2 - \upsilon_2^2)$

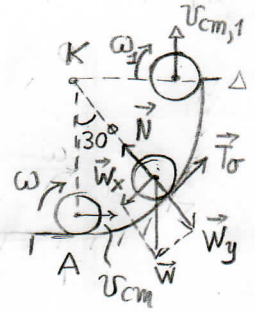
$= 1{,}08\cdot10^5 + 0{,}5\cdot10^4 + \frac{1}{2}\cdot10^3(-12) = (1{,}08 + 0{,}05 - 0{,}06)\cdot10^5 =$

$= 1{,}07\cdot10^5\ Pα$ (6μ)

Για το ύψος h_2: $P_2 = p_{ατ} + \rho g h_2 \Rightarrow h_2 = \frac{P_2 - p_{ατ}}{\rho g} \Rightarrow h_2 = \frac{0,07 \cdot 10^5}{10^4} \Rightarrow$

$h_2 = 0,7\,m$ (4μ)

Θέμα Δ | $m = \frac{10}{7}\,kg$, $r = 0,1\,m$, $v_{cm} = 6\,m/s$, $R = 1,5\,m$



Δ1) $\Sigma \vec{F}_x = m \cdot \vec{a}_{cm} \Rightarrow T_\sigma - mg\cdot n\mu\varphi = m \cdot a_{cm}$

$\Sigma \vec{\tau}_{c.m.} = I_{cm} \cdot \vec{a}_{γων} \Rightarrow -T_\sigma \cdot r = \frac{2}{5} m r^2 \frac{a_{cm}}{r}$ $\Big\}$ $\overset{+}{\Longrightarrow} -mg n\mu\varphi = \frac{7}{5} m \cdot a_{cm} \Rightarrow$ (7μ)

$\Rightarrow a_{cm} = \frac{-5 g n\mu\varphi}{7} \Rightarrow a_{cm} = \frac{-25}{7}\,m/s^2$, $\alpha_{γων} = \frac{a_{cm}}{r} = \frac{-250}{7}\,\frac{rad}{s^2}$

Δ2) $\frac{dL_s}{dt} = I_{cm} \cdot \alpha_{γων} = \frac{2}{5} m r^2 \cdot \alpha_{γων} = \frac{2}{5} \cdot 0,7 \cdot 0,01 \cdot \left(-\frac{250}{7}\right) = -0,1\,kgm^2/s$ (4μ)

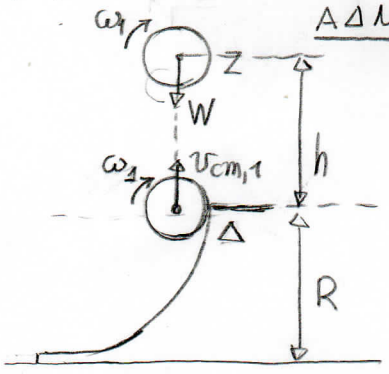
Δ3) $K_\Delta - K_A = W_W + W_{T_\sigma}^{0} + W_N^{0} \Rightarrow \frac{7}{10} m v_{cm,1}^2 - \frac{7}{10} m v_{cm}^2 = -mg(R-r) \Rightarrow$

$7 v_{cm,1}^2 = 7 v_{cm}^2 - 10 g (R-r) \Rightarrow v_{cm,1} = \sqrt{v_{cm}^2 - \frac{10}{7} g (R-r)} =$ (5μ)

$= \sqrt{36 - \frac{10}{7} \cdot 14} = \sqrt{16} = 4\,m/s$, $\omega_1 = \frac{v_{cm,1}}{r} = \frac{4}{0,1} = 40\,rad/s$

$L_s = I_{cm} \cdot \omega_1 = \frac{2}{5} \cdot 0,7 \cdot 0,01 \cdot 40 = 0,112\,kgm^2/s$ (2μ)

Δ4) $\Sigma \tau_{c.m.} = 0 \Rightarrow \omega_1 = 6\,rad/\theta$. (1μ)



ΑΔΜΕ Δ → Z

$\frac{1}{2} m v_{cm,1}^2 + \frac{1}{2} I_{cm} \omega_1^2 = \frac{1}{2} I_{cm} \omega_1^2 + mgh \Rightarrow$

$h = \frac{v_{cm,1}^2}{2g} = \frac{16}{20} = 0,8\,m$ (5μ)

$H = h + R = 0,8 + 1,5 = 2,3\,m$ (1μ)

* Η κινητική ενέργεια σφαίρας, που εκτελεί Κ.Χ.Ο. είναι $K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} \cdot \frac{2}{5} m \underbrace{r^2 \omega^2}_{v_{cm}^2} = \frac{1}{2} m v_{cm}^2 + \frac{1}{5} m v_{cm}^2 = \frac{7}{10} m v_{cm}^2$