***Ασκήσεις στη διάταξη των πραγματικών αριθμών***

 **Θωμάς Μαμάκος**

 **Μαθηματικός**

1. Αν $\left\{\begin{array}{c}α\leq 2β\\β<3\end{array} ν.δ.ο. α<6\right.$
2. Αν $α^{2}<β ν.δ.ο. β>0 , α^{2}-3β< $0
3. Αν $\left\{\begin{array}{c}α-β>1\\β\geq 0\end{array} ν.δ.ο. α>β , α>1 , \right.α^{2}-β^{2}>α+β$
4. Αν $\left\{\begin{array}{c}0<α\leq 2\\α+β=5\end{array} ν.δ.ο. 3\leq β<5 , αβ\leq 6 \right.$
5. Αν $\left\{\begin{array}{c}αβ\leq 1\\β>4\end{array} ν.δ.ο. α<\frac{1}{4}\right.$
6. Αν $\left\{\begin{array}{c}α^{2}\leq 2β\\β<2\end{array} ν.δ.ο. α<\right.2$
7. Αν -2 < α < 1 $ν.δ.ο.$ $α^{2}<4$
8. Αν $\left\{\begin{array}{c}α^{2}+β=2α\\β>0\end{array} ν.δ.ο. 0< α<\right.2, β\leq 1$
9. Αν $\left\{\begin{array}{c}α^{3}-β^{3}=1\\β>0\end{array} ν.δ.ο. α>1\right., α>β , αβ\left(α-β\right)<\frac{1}{3}$
10. Αν $\left\{\begin{array}{c}α^{2}-β^{2}=-1\\α>β\\α>0\end{array} ν.δ.ο. β<0\right.$
11. Αν $\left\{\begin{array}{c}α^{2}<4\\α+2>0\end{array} ν.δ.ο. α<\right.2$
12. Αν $\left\{\begin{array}{c}x+y\leq 3\\x>0\end{array} ν.δ.ο. y<\right.3 και x^{2}-2y^{2}\leq 18$
13. Αν $\left\{\begin{array}{c}α^{2}<β\\β<4\end{array} ν.δ.ο. α<\right.2 , αβ<8$
14. Αν $\left\{\begin{array}{c}α^{2}+αβ>1\\0< α<1\end{array} ν.δ.ο. α+β>1\right., β>0$
15. Αν $\left\{\begin{array}{c}α>2x\\x>α^{2}\end{array} ν.δ.ο. x>0, α>\right.0 , α<\frac{1}{2} , x<\frac{1}{4}$
16. Αν $\left\{\begin{array}{c}x>0\\y<1\\αβ\leq xy\end{array} ν.δ.ο. αβ<x\right.$
17. Αν $\left\{\begin{array}{c}α-β<1\\0< α<1\end{array} ν.δ.ο. α^{2}-αβ<1\right.$
18. Αν $\left\{\begin{array}{c}α^{2}+α=1\\α>0\end{array} ν.δ.ο. α<\right.1 , α^{2}<α, $ $α>\frac{1}{2}$ , $α^{2}<\frac{1}{2}$
19. Αν $\left\{\begin{array}{c}α+β>2\\α^{2}+β^{2}=4\end{array} ν.δ.ο. α>0, β>0\right.$
20. Αν $\left\{\begin{array}{c}α<2\\β<2\\αβ<2\end{array} ν.δ.ο. α+β<3\right.$
21. Αν $α+β>2 $ $ν.δ.ο.$ $α^{2}+β^{2}>2$ , $α^{4}+β^{4}>2$
22. Αν $α^{3}+α=1$ $ν.δ.ο.$ $α\ne 0 , α\ne 1 ,α\left(α^{2}+1\right)=1 , α>0 , α<1 $ , $α^{2}<\frac{1}{2}$
23. Αν $\left\{\begin{array}{c}α^{3}+β^{3}=-8\\β<-2\end{array} ν.δ.ο. α>0\right., α+β<0 , α^{2}<β^{2}$
24. Αν $\left\{\begin{array}{c}1\leq α\leq 2\\1\leq β\leq 2\end{array} ν.δ.ο. \frac{α}{β}+\frac{β}{α}\leq \frac{5}{2}\right.$
25. Αν $\left\{\begin{array}{c}α-β<2\\α^{2}-β^{2}=4\\α>0\end{array} ν.δ.ο. α>0, β>0 , α+β>2\right.$
26. Αν $\left\{\begin{array}{c}0<x+y\leq 1\\\frac{x}{y}>\frac{1}{2}\end{array} ν.δ.ο. x>0\right.,y>0 ,xy\leq \frac{1}{4} , y<\frac{2}{3}$
27. Αν $\left\{\begin{array}{c}x^{2}+y^{2}=2\\0<\frac{1}{x}+\frac{1}{y}\leq 2\\xy>0\end{array} ν.δ.ο. x>0\right., y>0 , \sqrt{2}<x+y\leq 2$
28. Αν $\left\{\begin{array}{c}x^{3}+y^{3}=8\\xy=2\end{array} ν.δ.ο. 0<x<2\right., 0<y<2 , x+y>2 , 4<x^{2}+y^{2}<6$
29. Αν $a^{2}+aβ+β^{2}\geq 3$ $ν.δ.ο.$ $a^{2}+β^{2}\geq 2$ και $a^{4}+β^{4}\geq 2$
30. Αν $a^{2}+aβ+β^{2}\leq 6$ $ν.δ.ο.$ $a^{4}+β^{4}\leq 72$
31. Αν $\left\{\begin{array}{c}α^{2}-β^{2}>4\\0<α+β<2\\\end{array} ν.δ.ο. β<0\right. , α-β<2, α>1$
32. Αν $x^{2}+y^{2}<1$ $ν.δ.ο.$ $x^{3}-3xy^{2}<1$