A NEW EXPLICIT RELATION FOR THE FRICTION COEFFICIENT IN THE DARCY-WEISBACH EQUATION

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ABSTRACT

The friction coefficient *f* in the Darcy – Weisbach equation for the linear losses of liquid flow under pressure in a closed cylindrical pipe, is approached by explicit relations that deviate from the Colebrook – White values up to 4% for certain Reynolds numbers' or relative roughness values. In the present paper we propose a relation, slightly more complex than the existing ones, which provides results that fall within the ±1% interval with respect to the implicit Colebrook – White equation, for all examined Reynolds numbers' and relative roughness values. All possible combinations of ten relative roughness values, from 10^{-5} to 0.05, as well as nineteen Reynolds numbers, from $4 \cdot 10^3$ to 10^8 were considered.

1. INTRODUCTION

The well-known Darcy-Weisbach equation that gives the linear friction losses in the case of liquid flow under pressure in a closed pipe:

$$\Delta h = f \frac{L}{D} \frac{U^2}{2g} \qquad (1)$$

includes friction factor *f*, which is historically determined by the Moody diagram [1]. As the use of the diagram is not very convenient for the determination of large numbers of friction coefficient values, many relationships were developed in order to facilitate the process. For very low values of the relative roughness, Nikuradse's implicit equation:

$$\frac{1}{\sqrt{f}} = 2 \cdot \log\left(\frac{\mathrm{Re} \cdot \sqrt{f}}{2.51} \right) \qquad (2)$$

and Colebrook's explicit equation [2]:

$$\frac{1}{\sqrt{f}} = 1.8 \cdot \log\left(\frac{\mathrm{Re}}{6.9} \right) \qquad (3)$$

were in use for decades. In the opposite case, of large relative roughness values, the Von Karman relation holds:

$$\frac{1}{\sqrt{f}} = 2 \cdot \log\left(\frac{3.7}{e/D} \right) \qquad (4)$$

where e is the absolute roughness and D the pipe's internal diameter. Colebrook equation [2], also known as the Colebrook – White equation is used worldwide covering the whole range of Reynolds numbers and relative roughness:

$$\frac{1}{\sqrt{f}} = -2 \cdot \log\left(\frac{e/D}{3.7} + \frac{2.51}{\mathrm{Re} \cdot \sqrt{f}} \right) \qquad (5)$$

but as its implicit scheme makes iterations necessary, many other equations were developed, which provide the determination of f explicitly. As will be shown in the following, these equations provide f with a precision ranging from 1% to more than 4% for some Reynolds numbers or relative roughness values. Obtaining a relation that would provide the friction coefficient with a higher accuracy, would be of interest, especially in cases of a large number of pipes, as, for example, in the process of the optimization of a large water supply network.

In the present paper, we propose a relation that fits values of the friction coefficient as given by the Colebrook-White equation for a wide range of Reynolds numbers and relative roughness. In the following, we remind the existing explicit relations, we propose a new relation and we compare relative errors.

2. THE EXISTING EXPLICIT RELATIONS

Among the various explicit equations used to approximate the friction coefficient, the Haaland equation is one of the mostly used [3]:

$$\frac{1}{\sqrt{f}} = -1.8 \cdot \log\left(\left(\frac{e/D}{3.7} \right)^{1.11} + \frac{6.9}{\mathrm{Re}} \right) \qquad (6)$$

which can be re-written as:

$$f = \frac{0.308642}{\left[\log\left(\left(\frac{e}{3.7 \cdot D} \right)^{1.11} + \frac{6.9}{\mathrm{Re}} \right) \right]^2} \qquad (7)$$

In order to check the accuracy of the values obtained for the friction coefficient by eq. (7) with respect to the values derived from eq. (5) after 10 iterations, we considered 10 values of the relative roughness *(e/D)* and 19 values of the Reynolds number *Re*, in all possible combinations. These *Re* and *(e/D)* values can be seen in table 1. For each pair of *(e/D,Re)* we calculated the relative error:

$$error = \frac{f_{Colebrook-White}}{f_{equation}} - 1 \qquad (8)$$

All errors referred to in the present, are relative errors, with respect to the Colebrook equation. That is, whenever "error" is referred in the following graphs, it is as follows: The values of f, were calculated from the Colebrook equation, after 10 successive approximations, resulting to a difference between successive values of less than 10^{-6}. The values of f were found and compared with values of the other equations mentioned in the following.

TABLE 1. Relative roughness and Reynolds number values used to check the accuracy of various relations.

e/D	***Re***
0.000001	4000
0.000005	8000
0.00001	$1 \cdot 10^4$
0.00005	$2 \cdot 10^4$
0.0001	$4 \cdot 10^4$
0.0005	$8 \cdot 10^4$
0.001	$1 \cdot 10^5$
0.005	$2 \cdot 10^5$
0.01	$4 \cdot 10^5$
0.05	$8 \cdot 10^5$
	$1 \cdot 10^6$
	$2 \cdot 10^6$
	$4 \cdot 10^6$
	$8 \cdot 10^6$
	$1 \cdot 10^7$
	$2 \cdot 10^7$
	$4 \cdot 10^7$
	$8 \cdot 10^7$
	10^8

Figure 1 shows the relative error when using the Haaland relation (equation 7). The relative error exceeds 1% for some relative roughness values in medium Re values (between $2 \cdot 10^4$ and 10^6).

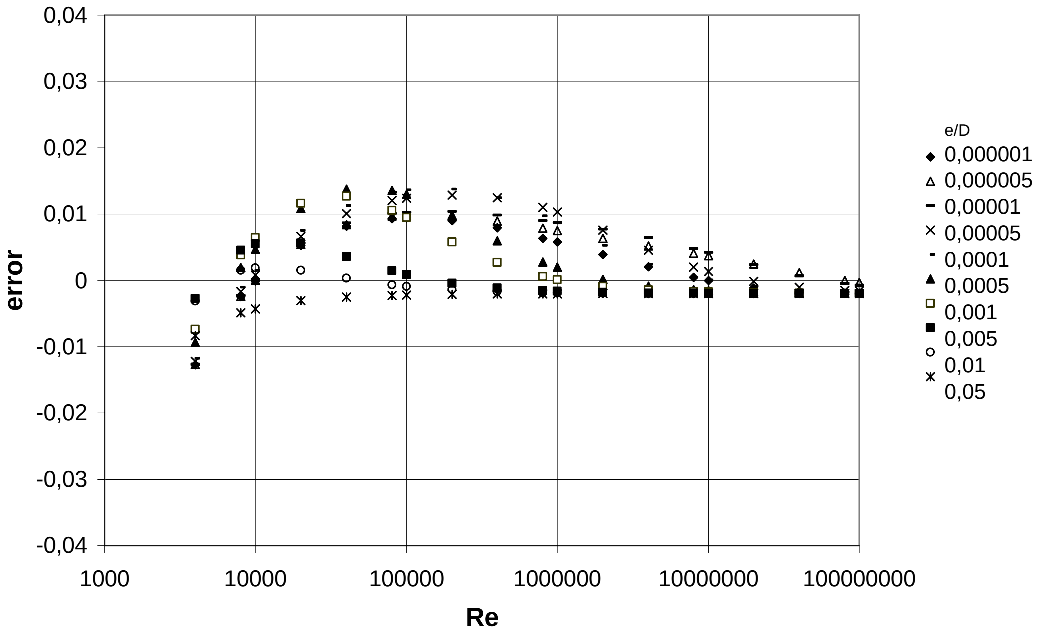

Figure 1. The relative error using the Haaland equation.

The second equation examined was the Swamme - Jain equation [4]:

$$f = \frac{0.25}{\left[\log\left(\frac{e}{3.7 \cdot D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \qquad (9)$$

which was proven to provide results that differ from those of Colebrook up to 3% (figure 2). The error is lower than 1% for all roughness values, only when $Re>8 \cdot 10^4$. In general, when relative roughness exceeds 10^{-3} and Reynolds number is inferior to 10^5, the Swamme - Jain equation overestimates f as much as 3% with respect to the Colebrook formula. Moreover, it seems in figure 2 that the relative error follows an exponential pattern with respect to Reynolds number.

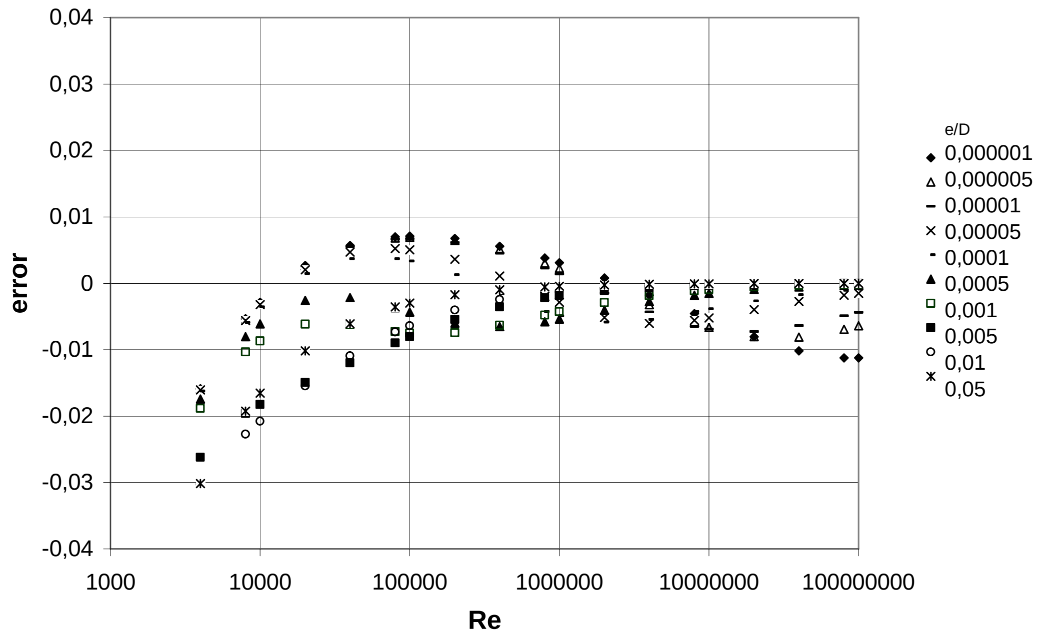

Figure 2. The relative error using the Swamme-Jain equation.

There have been more recent attempts for an accurate, explicit relation. Sonnad and Goudar [5, 6] presented an equation more complex than the Haaland equation, which was questioned as for its results, in the sense that the maximum error is restricted to a relatively small area characterized by low *Re* and small *(e/D)* [7].

Clamond [8], presented an algorithm with much more accurate results, but with the need of much more demanding computations. Very accurate results are also given by the method of Serghides [9], which nevertheless, demands a lot of computing, and more than one equation. Chen [10], proposed the following equation for all values of *Re* and *(e/D)*:

$$\frac{1}{\sqrt{f}} = -2 \cdot \log\left[\frac{e}{3.7065 \cdot D} - \frac{5.0452}{\mathrm{Re}} \cdot \log\left(\frac{1}{2.8257} \cdot \left(\frac{e}{D}\right)^{1.1098} + \frac{5.8506}{\mathrm{Re}^{0.8981}}\right)\right] \qquad (10)$$

This equation restricts errors from -0.22% to +0.47%, but it involves the dual appearance of both *(e/D)* and *Re*.

3. THE NEW EXPLICIT RELATION

In an attempt to find an explicit relationship for the friction factor f of the Darcy-Weisbach equation that should be accurate and as simple as possible at the same time, we noticed that error values tended to "bend" to negative values in an exponential way for Re lower than 10^6. In order to cope with that, we used a relation of the following form:

$$f = \frac{a_1 - a_2 \cdot (7 - \log \mathrm{Re})^4}{\left[\log\left(\frac{e}{a_3 \cdot D} + \frac{\alpha_4}{\mathrm{Re}^{a_5}}\right)\right]^2} \qquad (11)$$

where the four parameters α_i, were to be determined.

We optimized the above-mentioned parameters, by imposing in parallel that the error values of f for low *Re* and high relative roughness should be lower than 0.8%. This gave equation (12), where all error values, for any roughness and any *Re* examined, are lower than 0.8%.

$$f = \frac{0.2479 - 0.0000947 \cdot (7 - \log \mathrm{Re})^4}{\left[\log\left(\frac{e}{3.615 \cdot D} + \frac{7.366}{\mathrm{Re}^{0.9142}}\right)\right]^2} \qquad (12)$$

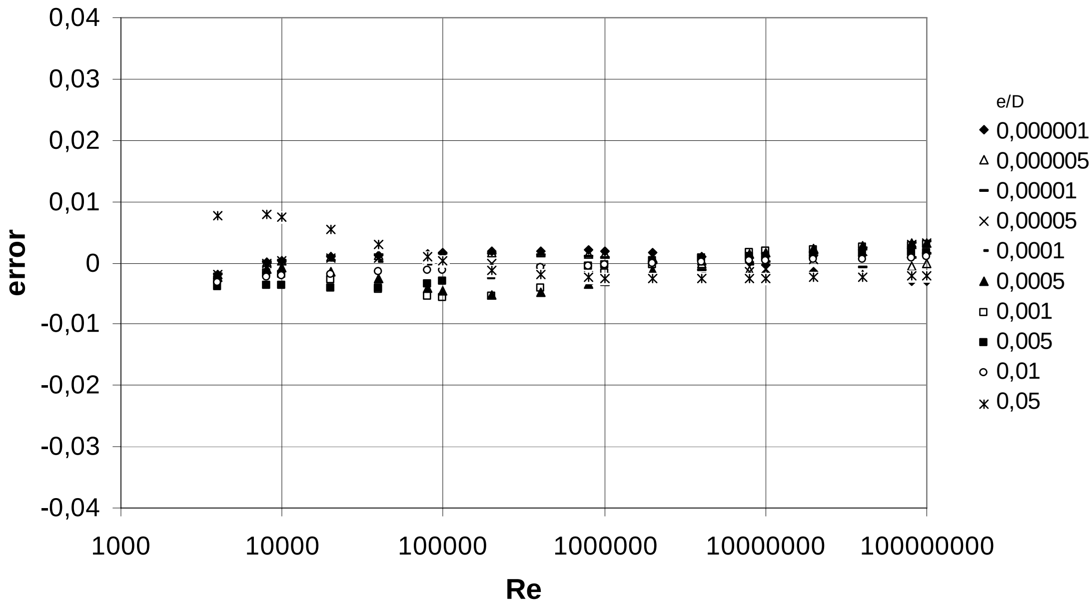

Figure 3. The relative error using the proposed equation.

As can be seen in figure 3, with the exception of very rough pipes (*(e/D)*>0.01) and low *Re*, errors remain within the range +0.2% to -0.6%. The only disadvantage, is the number of parameters in the nominator of the equation.

Figure 4 shows contour lines of f, according to equation (12), for the whole range of the parameters. Logarithms in this graph are decimal.

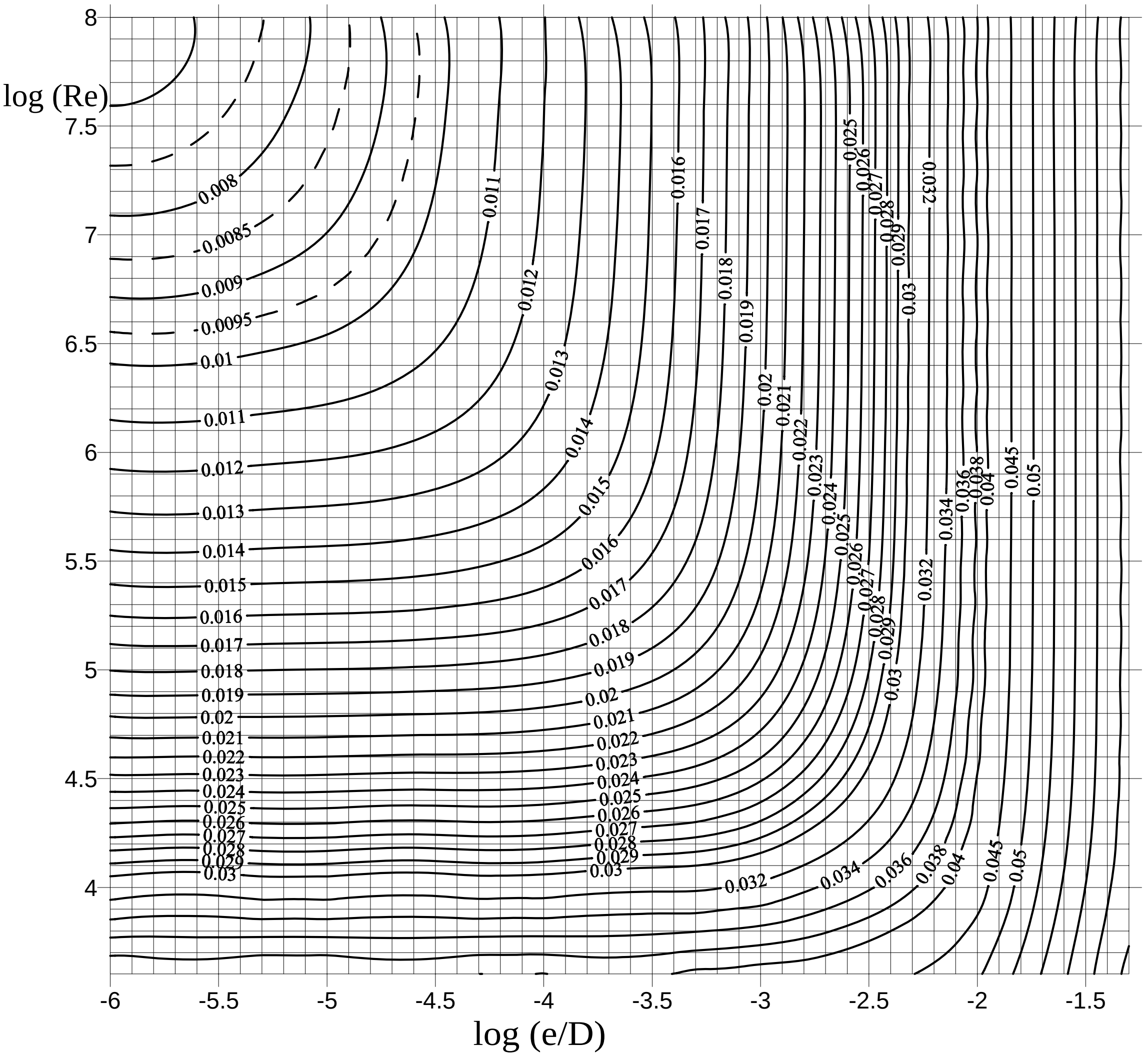

Figure 4. The friction coefficient in turbulent flow (Re > 4000), according to eq. 12.

4. DISCUSSION – CONCLUSIONS

In order to check out the regions where the results of our proposed equation (eq. 12) deviate the most from the respective values of Colebrook, we plotted the relative error as defined in eq. 8, in figure 5.

It is seen there that this error exceeds 0.5% in two regions: one in the upper left corner of the graph, where $Re > 10^7$ and $(e/D) < 10^{-5}$, and one in the lower right corner of the graph, where $Re < 10^4$ and (e/D) is close to 0.01. And even in these cases, the error remains at values lower than 0.8%.

The proposed equation is much less complex than Chen's equation (eq. 10), which also gives very good results. And it is much less complex than the algorithms of Serghides and Clamond. For this reason we believe that it can be used in order to facilitate calculations in cases where the estimation of a large number of friction coefficients is needed.

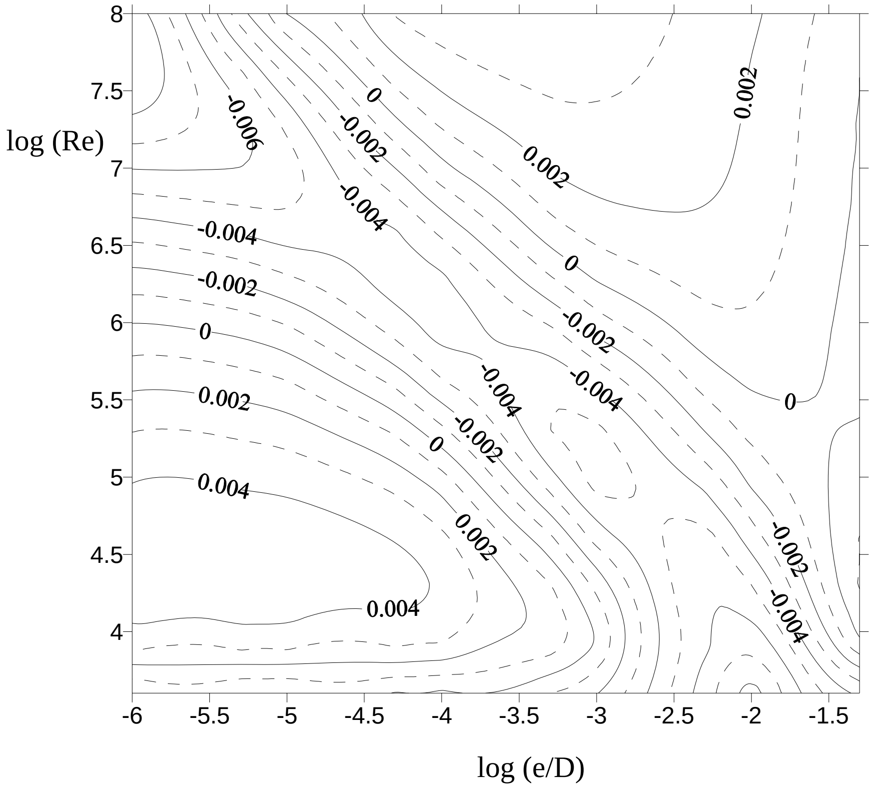

Figure 5. The relative error using the proposed equation.

Considering that the proposed equation is not significantly more complex than the existing ones, we suggest it can be used in order to facilitate friction coefficient calculations with better accuracy than with the existing relations.

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