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Statistical analysis and simulation of a hydrostatic force experimental device

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Abstract: - In the present paper, a statistical analysis of the experimental results of the hydrostatic force versus water depth is provided. The experiments were executed on a model setup in the Laboratory of Hydraulics in the Department of Civil Engineering of the Technical Educational Institute of Serres, Greece. The results of a total of 79 identical experiments are presented. Each experiment has concluded in a certain linear relation between depth and force. A statistical analysis was carried out, testing the hypothesis that the residuals of the experimental values from the respective theoretical parameter values are normally distributed. This hypothesis was found to be true at a level of significance of 0.1, leading to the conclusion that only random errors have occurred, thus excluding the systematic ones.

Key-words: - hydrostatic force, experimental device, statistical analysis, hypotheses testing

1 Introduction

Errors and uncertainties are present in every experimental observation. Knowing the precision of the measurement device and technique with which we measure a physical quantity is essential for the importance we can attribute to the results.

For the analysis of measurements of hydrostatic forces, a lot of works exist in the literature (ex. [1, 2, 3].

The purpose of the present paper is to examine a large number of experiments on hydrostatic forces and to test the hypothesis that no systematic errors occurred during these series of measurements.

In section 2 we present the experimental device used, and in section 3 its mathematical modeling. Section 4 gives the statistical analysis of the parameters and section 5 the hypothesis testing.

2 The experimental device

We present here a statistical analysis of results of experiments carried out in the Hydraulics Laboratory of the Department of Civil Engineering of the Technical Educational Institute of Serres, in the North of Greece. A total of 79 experiments were carried out in total. In each experiment N couples of the values of the water depth and the resulting hydrostatic force on a plane.

The purpose of each experiment is to determine the best values of the parameters α_0 and α_1, corresponding to the relation:

$$F_{\exp-i} = \alpha_0 + \alpha_1 \cdot y_i \quad (1)$$

where F_{exp-i} is the experimental value of the hydrostatic force and y_i the water depth. In this way, the equation of the simulation of the apparatus is extracted, with the least squares method.

The statistical analysis that follows, provides further data processing of the distribution functions of α_0 and α_1 as well as of the relative error between the theoretical values F_{th-i} and the improved experimental values F_{exp-i}' which are given by theory and eq. 1 respectively, as well as the check of statistical hypotheses on the kind of the errors of the measurements.

The Armfield F1-12 was used as experimental device, which measures the hydrostatic force exerted on a rectangular plane surface. This plane surface is one side of a torroid shaped body, partially immersed in water and supported at a pivot point (Fig. 1). This device thus includes [4]:

- An 7 liter open rectangular reservoir (Perspex).
- A torroid-shaped body, whose shape can be described as a rectangle which revolves for 90 degrees around an axis outside it.

- A metallic horizontal axis where the above-mentioned body is attached and which can revolve.
- A system of hanging weights on the end of the metallic axis. These weights balance the hydrostatic force.
- Systems to ensure that the bottom of the reservoir is horizontal and the axis parallel to the bottom.

As will be described later, the balance of two torques that come from the weights and the hydrostatic force respectively, results on the determination of the latter in function with the water depth.

This device operates in two different water profiles: deep (160 mm > y_i > 100mm) and swallow (y_i < 100mm). The deep profile was exclusively used in the experiments of the present paper. In each experiment, 20 pairs of values were measured (y_i – water depth in mm, m_i – weight mass in gr), covering the whole depth range.

The experiment advances by increasing gradually the water depth and measuring the mass necessary to counterbalance the momentum of the hydrostatic force.

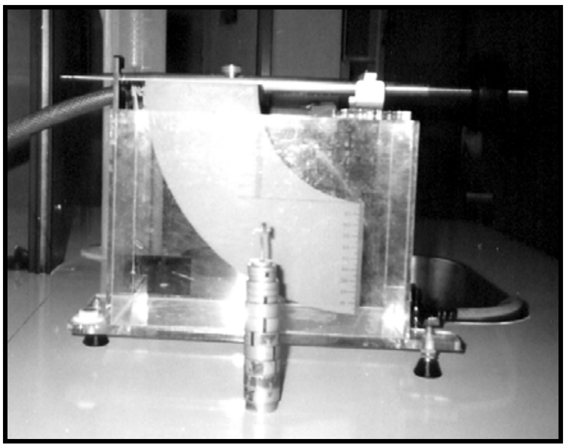

Fig. 1. The Armfield hydrostatic force experimental device.

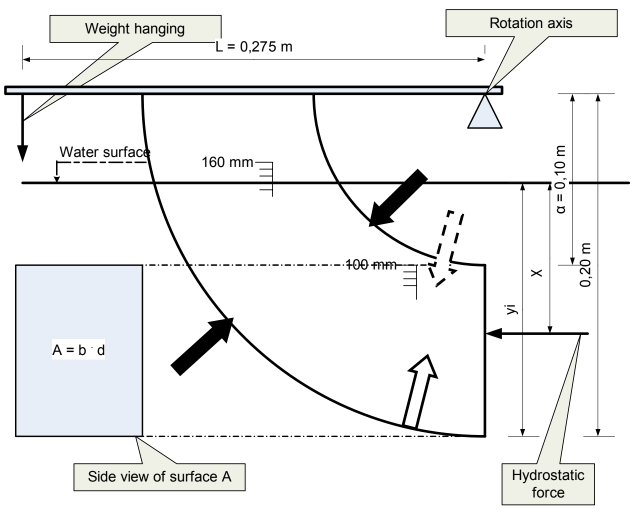

Fig. 2. The schematic representation of the device.

When water depth is increased, the following three forces are applied on the body:

a. Two forces on the side surfaces of the body, which are counterbalanced. They are not indicated in fig. 2.
b. Two forces on the convex and concave surfaces, which pass from the rotation axis and thus, they do not create any torques on the horizontal axis (black arrows in fig. 2).
c. A force that is applied on the vertical area A and that provokes a torque. The theoretical value of this force in function of the depth is:

$$F_{th-i} = \rho \cdot g \cdot A \cdot \left(y_i - \frac{d}{2} \right) \tag{2}$$

ρ the fluid's density, g the acceleration of gravity, A =b·d the area of the surface the force is applied to.

3 The mathematical model

The experimental values of the hydrostatic force F_{exp-i} deviate from F_{th-i} (eq. 2) because of random errors and uncertainties during measuring. Systematic errors cannot exist, because the device is calibrated twice a year. On the other hand, the hypothesis that only random errors existed during our measurements is examined in the following (section 5).

The experimental value F_{exp-i} results from the counterbalance of the torque of the hydrostatic force described earlier and the one of the weight of the hanged mass. The equation that gives this experimental force F_{exp-i} in function of the depth (y_i) and the weight (m_i) is:

$$F_{\exp-i} = \frac{m_i \cdot g \cdot L}{\alpha + \frac{d}{2} + \frac{d^2}{12 \cdot \left(y_i - \frac{d}{2} \right)}} \tag{3}$$

where (fig. 2) m_i the mass of the hanged weights in kg, L = 0.275 m the length of the horizontal axis, α = 0.10 m the radius of the concave surface of the body.

For the determination of the optimal straight line that represents each experiment's data the least square method was used [5] to best fit the "cloud" of the points (y_i, F_{exp-i}).

By solving the system of normal equations:

$$\begin{aligned} N \cdot \alpha_0 + \sum_{i=1}^{N} y_i \cdot \alpha_1 &= \sum_{i=1}^{N} F_{\exp-i} \\ \sum_{i=1}^{N} y_i \cdot \alpha_0 + \sum_{i=1}^{N} y_i^2 \cdot \alpha_1 &= \sum_{i=1}^{N} \left(y_i \cdot F_{\exp-i} \right) \end{aligned} \tag{4}$$

one obtains the coefficients α_0 and α_1 of eq. 1. The theoretical values of these coefficients are:

$$\alpha_0 = -3.67875 \text{ N} \quad \text{and} \quad \alpha_1 = 73.575 \text{ N/m} \tag{5}$$

If then one substitutes in eq. 3 the depth values, one obtains the improved experimental values $F_{exp\text{-}i}'$. These values can be correlated with the theoretical values $F_{th\text{-}i}$, in a way to find with a new application of the least square method, the linear relationship that relates them, considering $F_{th\text{-}i}$ as independent variable and $F_{exp\text{-}i}'$ as dependent variable (eq. 6) as well as the experimental error (eq. 7):

$$F_{\exp-i}' = \alpha_0' + \alpha_1' \cdot F_{th-i} \tag{6}$$

$$\text{experimental error \%} = \left(\frac{F_{th-i}}{F_{\exp-i}'} - 1 \right) \cdot 100 \tag{7}$$

In the statistical analysis that follows, we use the experimental results on coefficients α_i and errors as data. We analyze statistically the values found for α_0 και α_1, the mean error of the measurements, and the parameters α_0' και α_1', which come out in the last phase of each experiment from the correlation of the improved experimental values with the theoretical ones. The expected values for these two last parameters are 0 and 1 respectively, since a parity of $F_{th\text{-}i}$ with $F'_{exp\text{-}i}$, is expected.

4 Statistical analysis

The measurements with the device are influenced only by random errors, as previously explained, which can be quantified and analyzed statistically. Below (section 5) we examine through the results the statistical hypothesis that the residuals, the deviations of the results from the respective theoretical values follow the normal distribution and are thus due only to random errors [6, 7].

In table 1, the statistical values of the 79 experiments are shown. Indicative histograms of values of coefficient α_0 and of the error are shown in fig. 3 and 4.

Table 1. Statistical values of the parameters.

	a_0 (N)	a1 (N/m)	a_0'	a_1'
Mean	-3,793	73,869	-0,030	1,0058
Error	0,0402	0,3123	0,0222	0,0050
Standard deviation	0,3577	2,7756	0,1977	0,0445
Variance	0,1279	7,7039	0,0390	0,0020
Kyrtosis	1,0136	4,8511	3,6458	5,2528
Skewness	-0,6731	1,0407	-1,0450	1,7570

The mean values for α_0 and α_1 (-3,79 N και 73,87 N/m respectively) are very close to the theoretical values of these parameters (eq. 5). The mean values of α_0' and α_1' were found –0.0303 and 1.0057 respectively, instead of the theoretical 0 and 1. The mean value of the mean experimental error (between theoretical and improved experimental values) is 3.26%, which is satisfactory.

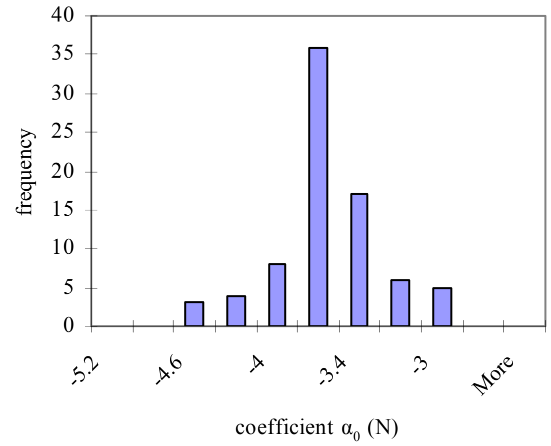

Fig. 3. Histogram of the distribution of coefficient α_0.

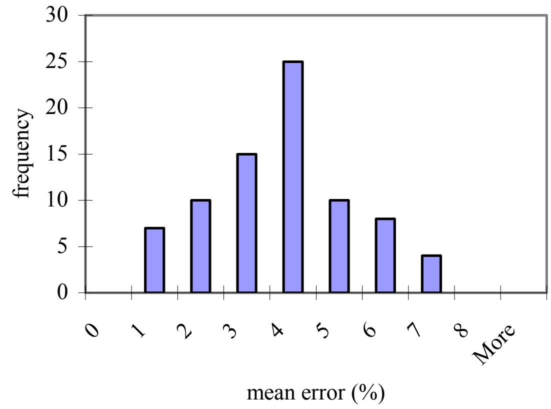

Fig. 4. Histogram of the distribution of values obtained for the error (eq. 7).

In table 2 the confidence intervals of α_0, α_1, α_0', α_1' are shown, in various confidence levels. These confidence levels can serve as a reference for the realization of future experiments with the same device.

Table 2. Confidence intervals of the parameters α_0, α_1, α_0', α_1'.

Conf. level	**α_0 (N)**		**α_1 (N/m)**	
	from	**to**	**from**	**to**
95%	-3,76	-3,60	73,2	74,5
99%	-3,78	-3,57	73,1	74,7
99,73%	-3,80	-3,56	72,9	74,8
Conf. level	**α_0' (N)**		**α_1' (N/m)**	
	from	**to**	**from**	**to**
95%	-0,044	0,044	0,990	1,010
99%	-0,057	0,057	0,987	1,013
99,73%	-0,067	0,067	0,985	1,015

5 Hypothesis testing

By normalizing the statistical data for the parameters α_0 and α_1, with their respective mean values and standard deviations, we obtain [8]:

$$z(i) = \frac{x(i) - \mu}{\frac{\sigma}{\sqrt{n}}} \tag{8}$$

where z(i) is the normalized value of α_0 and α_1, x(i) the value of parameters α_0 and α_1 of experiment i, μ the corresponding mean value of the parameters, σ their standard deviation and n the number of the experiments.

These values can be compared with the respective curves of normal distribution [9]. This comparison is given in fig. 5, where for every normalized value of observations of α_0 and α_1, there is the respective theoretical value of the normal distribution. For every normalized experimental value z of parameters α_0 and α_1 we calculated the theoretical probability P_{th} for each measurement to be lower than z according to normal distribution [$P_{th}(Z<z)$], and the corresponding experimental probability $P_{exp}(Z<z)$ [10, 7]. The theoretical value of the probability is given by:

$$P_{th}(Z < z) = \frac{1}{2}(1 + erf(\frac{z}{\sqrt{2}})) \tag{9}$$

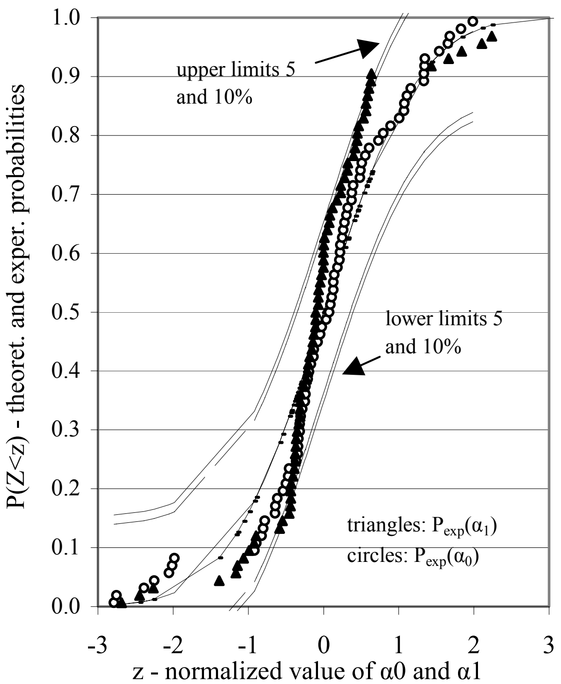

Fig. 5. The Kolmogorov-Smirnov goodness of fit test for the probability distribution of parameters α_0 and α_1.

Moreover, in fig. 5 the limits of the deviations (P_{exp}-P_{th}) for significance levels 5 και 10% are shown. The agreement of the data with the theoretical curves is satisfied in both levels for the results of parameter α_0, while for α_1 only in the 10% level. It is obvious that this check would be satisfied for α_0 in significance levels even lower than 5%. Considering the large number of experiments, these levels can be characterized as quite satisfactory. The differences between experimental and theoretical values of the probability are such that allow us to accept the hypothesis that errors are randomly distributed.

6 Conclusion

In the present paper we present the results of 79 experiments on the relation of the hydrostatic force with depth. In every experiment, the values of the parameters of this linear relationship were determined. The mean values of the basic parameters were found very close to the theoretical ones and the mean error was 0.0326.

The statistical hypothesis of the normal distribution of the deviations of the experimental parameter values from the theoretical ones was tested, in order to check whether systematic errors could exist. This hypothesis was accepted for a significance level of 10% for the first parameter and much lower than 5% for the second.

The methodology followed can be applied to other experimental devices in our Laboratory, such as linear losses and local losses devices, Venturi devices and electromagnetic flow rate devices.

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