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Estimation of Evapotranspiration Using Fuzzy Systems and Comparison With the Blaney-Criddle Method

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Abstract: In this research, the possibility of predicting potential evapotranspiration using fuzzy logic theory with temperature as input is studied. The fuzzy logic model is being trained using a series of temperature and evapotranspiration values. The results are being compared to measured values and with the ones obtained using the Blaney-Criddle method. The results show a high efficiency in calculating and predicting the potential evapotranspiration values.

Key words: Blaney-Criddle, evapotranspiration, fuzzy rules, prediction

INTRODUCTION

The most common method to deal with the uncertainties was probability theory, until 1965, when Zadeh introduced the fuzzy set theory. Fuzzy logic is an effective tool for handling the ambiguity and uncertainty of the real world systems. The Fuzzy Rule-Based (FRB) systems or Fuzzy Inference Systems (FIS) originate from fuzzy logic and the fuzzy set theory in general. FRB systems provide an effective way to capture the approximate nature of the real world processes, due to the rule formulation. The relationships can be described verbally instead of using mathematical equations. The input and output variables are related using fuzzy IF-THEN rules, where IF relates to the vector of fuzzy premises and THEN to the vector of the consequences, which has the form of a fuzzy set as well. The linguistic formulation of the rules resembles the human reasoning, since it is based on IF-THEN expressions and this is why fuzzy logic is such an effective tool in describing ambiguity. FRB models provide answers to problems where the procedures are complex and difficult to define mathematically.

In recent years, many applications using fuzzy logic theory appeared, since, it is an alternative and effective tool for studying complex phenomena. Fuzzy logic models can give answers to practical problems, without being time consuming. Fuzzy rule systems have been used successfully in reservoir management (Panigrahi and Mujumdar, 2000), rainfall-runoff problems (Nayak *et al*., 2005; Yu and Chen, 2005) and in parameters of groundwater flow (Moutsopoulos *et al*., 2005).

In the present study, an attempt is made to apply FRB to simulate evapotranspiration values using temperature data.

FUZZY LOGIC

In Boolean logic the boundaries of a set are clearly defined and it is evident whether an object belongs to a set or not. It is described by a binary function, the characteristic function, taking the value 0 when an element x does not belong to the set A and the value 1 when it does. On the contrary, in fuzzy logic, it is possible for elements to belong partially to a set. If X is a collection of objects denoted, generally, by x, then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs:

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$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) \middle| x \in X \right\} \quad (1)$$

where, $\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership of x in $\tilde{A}$. The degree to which an element belongs to a fuzzy set is shown by the membership function, which takes values within the [0, 1] interval.

FUZZY RULE SYSTEMS

Fuzzy System Function

A fuzzy system R is defined as a collection of rules consisting of fuzzy premises $A_{i,k}$ with membership functions $\mu_{Ai,k}$ and fuzzy responses $B_{i,k}$ having also the form of a fuzzy set. A fuzzy rule has the following general form:

$$\text{If } a_1 \text{ is } A_{i,1} \otimes a_2 \text{ is } A_{i,2} \otimes \ldots\ldots \otimes A_K \text{ is } A_{i,K} \text{ then } B_i$$

where, a_k the k input value, A_i the fuzzy set corresponding to the fuzzy premise and B_i the fuzzy set of the response.

All rules use the same variables as fuzzy premises and the same variable as fuzzy response. A fuzzy rule is composed of fuzzy sets connected together with logical operators. The most usual ones are the AND, OR represented by the symbol $\otimes$.

In fuzzy systems, the partial fulfillment of a rule is possible. The degree to which a rule is true is called Degree of Fullfillment (DOF). Zero DOF value denotes no fulfillment of the rule and unitary value means that the rule is fully applicable. The values within the interval [0, 1] show the degree of applicability of the rule. If there is more than one fuzzy premise, then the membership values should be combined, in order to obtain the DOF. The AND operator corresponds to the union of the fuzzy sets and the OR operator to the intersection. AND operator is interpreted using a t-norm and OR is interpreted using a t-conorm. There is a great variety of t-norms and co-norms suggested in the literature (Zimmermann, 1985). The most usual ones are the product inference and the min-max method. The first one accounts for the fulfilment of all arguments in contrast to the min-max method where only the limiting or extreme argument is considered (Bárdossy and Duckstein, 1995).

AND operator

$$\nu_i = \prod_{k=1}^{K} \mu_{Ai,k}(a_k) \quad (2)$$

where, A_i is the fuzzy set of premise i, a_k the k input value and μ the membership function

OR operator

$$\nu(A_1 \text{ OR } A_2) = \mu_{A1}(\alpha_1) + \mu_{A2}(\alpha_2) - \mu_{A1}(\alpha_1) * \mu_{A2}(\alpha_2) \quad (3)$$

where, A_1 and A_2 are the fuzzy sets corresponding to the premises, μ the Degree of Fulfillment of the rule and α_1 and α_2 the input values.

Overlap between premise membership functions is necessary in order to reduce the error sensitivity. As shown in Fig. 1 overlap exists when the domains between neighboring fuzzy numbers partially cover each other. This means that when a crisp number is imported to the system, it belongs partially to more than one fuzzy numbers. Each premise fuzzy set corresponds to a different rule.

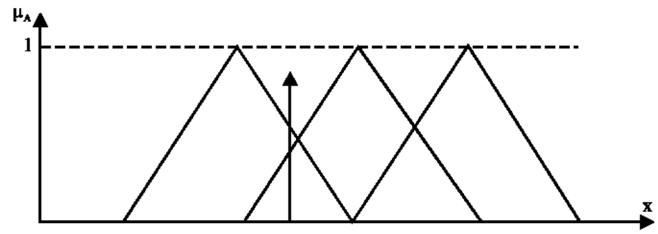

Fig. 1: Fuzzy numbers with overlap

When a crisp number belongs to an overlapping section then there are as many rules triggered as the fuzzy sets belonging to the particular section. Crisp values are assigned using a training set (to train the rules) and a verification set (to test model performance).

It is obvious that in this case every rule has a different DOF and leads to a different response for the same inputs. All fuzzy responses are being aggregated to a fuzzy set which results from all the fuzzy premises and takes into account also the DOFs. There is a number of methods suggested to aggregate fuzzy responses. The most usual ones are the maximum combination method, the minimum combination method and the additive combination method (Bárdossy and Duckstein, 1995).

The fuzzy set obtained from the above procedure is necessary to be replaced by a crisp number. This number must be the most representative of the fuzzy set, especially when a solution to practical problems is sought. The procedure in which the fuzzy set obtained from the rule responses is being transformed into a crisp number is called defuzzification. There is a plethora of defuzzification methods. The most usual one is the fuzzy mean method. This is the number for which the left part of the membership function is in equilibrium with the right side. This yields (Bárdossy and Duckstein, 1995):

$$\int_{-\infty}^{M(B)} (M(B) - t)\mu_B(t)\,dt = \int_{M(B)}^{+\infty} (t - M(B))\mu_B(t)\,dt \tag{4}$$

where, M(B) the fuzzy mean of the response B, t the input data and μ_B the membership function of the fuzzy set B.

Rule Construction

Rule construction is a procedure where available information and data are transformed into fuzzy rules. The rules can be constructed either empirically or by using available data. Usually, the procedure is quite difficult, due to the relationships between the variables that cannot be easily understood. In these cases a training set is used. A training set consists of input and output data. There are many techniques suggested to utilize the training set and export the fuzzy rules. The most common ones are (Bárdossy and Duckstein, 1995):

- Counting and weighted counting algorithm.
- Least squares algorithm.
- ANFIS algorithm (Adaptive Neuro-Fuzzy Inference System), (Jang, 1993). This method has similar function with artificial neural network and adapts the membership function parameters to minimize the system error.

In order to measure the system performance it is essential to use a verification set. The verification set consists also of input and output data. Input data are imported into the rule system and the responses are compared with the measured output values.

THE BLANEY-CRIDDLE METHOD FOR POTENTIAL EVAPOTRANSPIRATION

Blaney-Criddle suggested the following empirical formula:

$$E_P = K\frac{32+1.8T}{3.94}P \quad \left(\text{mm month}^{-1}\right) \tag{5}$$

where, E_p the potential evapotranspiration, K is a crop coefficient, T the mean monthly temperature (°C) and P the daylight percentage, given from tables.

The above empirical formula is the first one proposed by Blaney-Criddle (1950). A more precise calculation is possible by using emendatory coefficients so that more factors are taken into account. It was found that the Blaney-Criddle method approaches efficiently the results obtained by using Penman method to calculate pan evaporation.

The above method was modified (Doorenbos and Pruitt, 1977) resulting in the following formula:

$$E_p = a+b\lfloor p(0.46T+8.16)\rfloor \text{ mm day}^{-1} \tag{6}$$

Where:

$$a = 0.0043\,(RH_{min}-(n/N)-1.4) \tag{7}$$

where, RH_{min} the minimum relative humidity over a period of a month, n/N the relative daylight duration, T the mean monthly temperature, b a parameter depending on n/N and the wind speed, p mean daily percentage of annual daytime hours. Owing to data shortage, Eq. 5 was used in this application.

APPLICATION OF FUZZY LOGIC TO ESTIMATE EVAPOTRANSPIRATION

The feasibility of predicting the evapotranspiration values of Kalamoto meteorological station was studied using fuzzy logic theory. Kalamoto station (220 m a.m.s.l) is located in North Greece, into Mygdonia hydrological basin. The available data concern the daily evapotranspiration values, covering the period from March 2002 to February 2004. The results are compared with those given by the Blaney-Criddle method.

Rule System

The fuzzy model operates based on If-Then rules, where, If relates to the vector of fuzzy premises regarding the mean weekly temperature data of Kalamoto station. The Then relates to the vector of fuzzy response, which is the mean weekly evaporation. Rule construction was implemented using the ANFIS technique, contained to MATLAB. The model was executed using mean weekly temperature in combination with other fuzzy premises, like relative humidity and relative daylight duration. The results did not result to remarkable improvement and finally only one fuzzy premise was used. This means that there was no use of the operators AND, OR at the final fuzzy model and no use of a t-norm (min-max, product, etc.).

Several runs were executed in order to find the most efficient number of membership functions. The model seemed to approximate the observed values quite well when using 15 triangular fuzzy numbers. The procedure of premise membership function construction is highly subjective and is

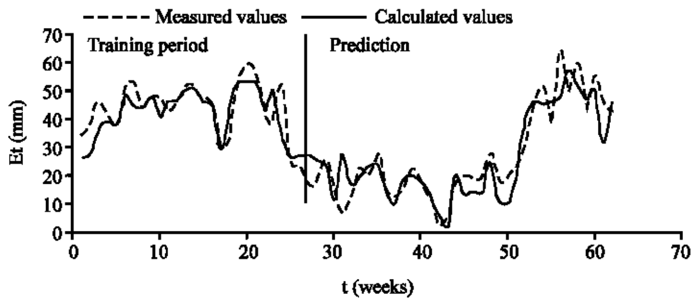

Fig. 2: Measured and calculated with the fuzzy logic model mean weekly evaporation values

implemented using the trial and error method in order to find the functions that represent the system efficiently. Fuzzy premise membership functions cover the entire observation set. The overlap between fuzzy premises at the trials was 50%.

Training Period

The entire set of observations is divided into a training set and a validation one. The training set is used when the rules cannot be directly formulated by the experts and is used to construct them. The training set consists of the mean weekly values of temperature at Kalamoto station and the mean weekly values of evaporation at the same station. The training period starts April 2002 and ends on February 2003. This means that it consists of 44 values, regarded enough for the rule system construction with one fuzzy premise.

Verification Period

The verification period is covering the period from March 2003 to August 2003 also consisting of the mean weekly values. The input is imported into the rule system and obtains the mean weekly evaporation values. The outputs are being compared to the measured evaporation values of Kalamoto station. In Fig. 2, the observed and the calculated values using the fuzzy logic model are shown. The results were also transformed to mean monthly values and compared with the ones obtained using the Blaney-Criddle method for potential evapotranspiration. Model performance is measured using the correlation coefficient and the reduced mean square error:

$$\text{Reduced MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{Y_{obs} - Y_{calc}}{Y_{obs}}\right)^2 \tag{8}$$

where, Y_{obs} is the observed value, Y_{calc} the one calculated from the model.

The reduced mean square error between mean weekly observed evaporation values and calculated using the fuzzy logic model is 0.042 and the correlation coefficient is 0.9146 for the training period. For the prediction period the reduced MSE is 0.0582 and the correlation coefficient is 0.8877. The use of only one fuzzy premise means that there was no use of an AND, OR operator and no use of a t-norm.

The results were also converted to mean monthly values and compared to the respective values computed using the Blaney-Criddle method. The reduced mean square error between mean monthly-observed evaporation values and calculated using fuzzy logic is 0.0294 and the correlation coefficient is 0.9673. The reduced mean square error between mean monthly-observed evaporation values and those calculated using the Blaney-Criddle method is 0.1107 and the correlation coefficient 0.902 (Fig. 3).

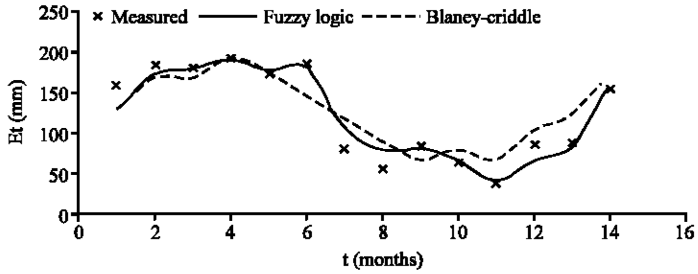

Fig. 3: Counted and calculated mean monthly evapotranspiration values using the fuzzy logic model and the Blaney- Criddle method

CONCLUSION

In this research, the capacity of fuzzy logic theory to be used for evaporation prediction was investigated. The model was constructed using ANFIS method and fifteen triangular fuzzy numbers. The Fuzzy logic model seems to have the ability to reproduce the evaporation values quite well. The construction and execution of the model are not time consuming and they can be used for prediction too. The only requirement involves the existence of a small series of measured temperature and evaporation values. The rule system is constructed in order to be used for each case separately. On the contrary conventional models require more parameters to be taken under consideration and follow the same methodology for all regions and climate conditions.

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